Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 1

Exercise 1

Compute $\int_D (2\,x\,y)\,dx\,dy\,dz$ for D=
$\{6\,y^9 \le x^3\,z^3 \le 11\,y^9,\ 4\,y^6 \le x^4\,z^2 \le 8\,y^6,\ 9 \le x^9\,y^2\,z^7 \le 18,\ x>0,\ y>0,\ z>0\}$

1) 1.70563

2) 1.60563

3) 0.00563081

4) −0.994369

5) −0.894369

Exercise 2

Compute the mean curvature for $X(u,v)=$
$\{5\,u+8\,v,\ -2\,u-3\,v,\ -3+4\,u^2-12\,v+2\,v^2+u\,(-7+3\,v)\}$ at the point $(u,v)=(-4,-4)$.

1) $H(-4,-4) = -4.85559$

2) $H(-4,-4) = -2.5043$

3) $H(-4,-4) = 3.07185$

4) $H(-4,-4) = 8.59437$

5) $H(-4,-4) = -0.000531549$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 9\,x & 0\le x\le 1 \\ -\frac{9\,x}{\pi-1}+\frac{9}{\pi-1}+9 & 1\le x\le \pi \end{cases} & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$
and the moment $t=1.$ by means of a Fourier series of order 11 .

1) $u(1,1.) = 3.79204$

2) $u(1,1.) = -4.85559$

3) $u(1,1.) = -5.85438$

4) $u(1,1.) = -7.79943$

5) $u(1,1.) = 0.000734452$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 2

Exercise 1

Compute $\int_D (2y + z^3)\,dx\,dy\,dz$ for D=
$\{3x^3 y \le 1 \le 11 x^3 y,\ 2x^5 \le z^4 \le 10 x^5,\ 2y^4 \le x^7 z \le 6 y^4,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.0204574

2) −0.879543

3) −0.779543

4) 1.72046

5) 1.92046

Exercise 2

Compute the mean curvature for X(u,v) =
{Cos[u] (3 + 2 Cos[v]) + 2 (3 + 2 Cos[v]) Sin[u] − Sin[v], 2 (3 + 2 Cos[v]) Sin[u] − Sin[v],
−((3 + 2 Cos[v]) Sin[u]) + Sin[v]} at the point (u,v) = (2, 2).

1) H(2, 2) = 5.29118

2) H(2, 2) = 3.55707

3) H(2, 2) = −8.77251

4) H(2, 2) = 2.02416

5) H(2, 2) = 0.0585632

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-3)\,x\,(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = 2\,(x-3)\,(x-2)\,x^2\,(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x = 2
and the moment t = 0.3 by means of a Fourier series of order 12.

1) u(2, 0.3) = 5.29118

2) u(2, 0.3) = −4.83318

3) u(2, 0.3) = 6.10633

4) u(2, 0.3) = 3.55707

5) u(2, 0.3) = −7.56929

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 3

Exercise 1

Given the function

$f(x, y, z) = -7 + 2x - x^2 + 6y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.28198, -4.71191, ?}

2) We have a minimum at {1, 3, ?}

3) We have a minimum at {-0.78198, -4.81191, ?}

4) We have a minimum at {0.0180197, ?, -1.38426}

Exercise 2

Consider the vectorial field $F(x, y, z) = (6xy^2, 6x^2y + 3, 0)$. Compute the potential function for this field whose potential at the origin is 5 .

. Calculate the value of the potential at the point p= (6 , 0 , -3) .

1) 5 2) $-\frac{5}{2}$ 3) $\frac{21}{2}$ 4) 7

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = -2(x-4)(x-2)(x-1)x^2 & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 3 and the moment t= 0.2 by means of a Fourier series of order 11 .

1) u (3 , 0.2) = -1.51279

2) u (3 , 0.2) = 2.36135

3) u (3 , 0.2) = 14.79

4) u (3 , 0.2) = -3.25395

5) u (3 , 0.2) = -3.51941

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 4

Exercise 1

Given the function

$f(x,y,z) = -8 + 6x - x^2 + 4y - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{16} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {2.12362, ?, 0.796357}

2) We have a maximum at {2.38907, ?, -0.796357}

3) We have a maximum at {2.65452, ?, 0.}

4) We have a maximum at {3, 2, ?}

Exercise 2

Consider the vectorial field $F(x,y,z) = (6x - 2xy^2, -2x^2y + 3yz^2\sin(yz) - 3z\cos(yz), 3y^2z\sin(yz) - 3y\cos(yz))$. Compute the potential function for this field whose potential at the origin is -3.

. Calculate the value of the potential at the point $p = (0, -8, -7)$.

1) $291 - 168\,\mathrm{Cos}[56]$ 2) $-3 - 168\,\mathrm{Cos}[56]$ 3) $-\frac{3999}{10} - 168\,\mathrm{Cos}[56]$ 4) $\frac{4233}{10} - 168\,\mathrm{Cos}[56]$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \leq t \\ u(x,0) = -(x-3)^2(x-2)(x-1)x & 0 \leq x \leq 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.6$ by means of a Fourier series of order 8.

1) $u(1, 0.6) = 3.87842$

2) $u(1, 0.6) = -0.45001$

3) $u(1, 0.6) = -1.57575$

4) $u(1, 0.6) = 2.16178$

5) $u(1, 0.6) = 2.55131$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 5

Exercise 1

Given the system

$-2y^2 - x u_5 == -62$
$3y^2 u_1 + 3x u_5^2 == -294$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-3, 5, -2, 4, 3, 2, -4)$. Compute if possible $\frac{\partial y}{\partial u_4}(-2, 4, 3, 2, -4)$.

1) $\frac{\partial y}{\partial u_4}(-2, 4, 3, 2, -4) = 0$

2) $\frac{\partial y}{\partial u_4}(-2, 4, 3, 2, -4) = 3$

3) $\frac{\partial y}{\partial u_4}(-2, 4, 3, 2, -4) = 1$

4) $\frac{\partial y}{\partial u_4}(-2, 4, 3, 2, -4) = 2$

5) $\frac{\partial y}{\partial u_4}(-2, 4, 3, 2, -4) = 4$

Exercise 2

Compute the Gauss curvature for X(u,v) = {-Cos[u] (2 + Cos[v]) + 2 (2 + Cos[v]) Sin[u], -Cos[u] (2 + Cos[v]) + (2 + Cos[v]) Sin[u] + 3 Sin[v], Sin[v]} at the point (u,v) = (3, 3).

1) K (3, 3) = 5.50931

2) K (3, 3) = -5.36535

3) K (3, 3) = -0.000781798

4) K (3, 3) = -5.11023

5) K (3, 3) = 0.926311

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=(x-3)\,(x-2)\,x\,(x-\pi)^2 & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 1. by means of a Fourier series of order 10 .

1) u(2 , 1.) = -1.09012

2) u(2 , 1.) = 4.59345

3) u(2 , 1.) = -8.57426

4) u(2 , 1.) = 5.50931

5) u(2 , 1.) = 0.0898754

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 6

Exercise 1

Compute $\int_D (x^2 + z^2)\, dx\, dy\, dz$ for D=

$\{7 x^3 y^8 z \le 1 \le 9 x^3 y^8 z,\ 7 x^4 y^7 \le z^2 \le 9 x^4 y^7,\ 8 x^4 y^7 z^2 \le 1 \le 11 x^4 y^7 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.000432734

2) −1.09957

3) −0.299567

4) 1.40043

5) −1.49957

Exercise 2

Consider the vectorial field $F(x, y, z) = (2 y z \cos(y z) - 1,$
$-z (2 x y z + y z) \sin(y z) + (2 x z + z) \cos(y z) - 3,\ (2 x y + y) \cos(y z) - y (2 x y z + y z) \sin(y z)$
$)$. Compute the potential function for this field whose potential at the origin is 0 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) −8.76023 2) −3.96023 3) −1.56023 4) −5.36023

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = -3 (x-5)(x-4)(x-1) x^2 & 0 \le x \le 5 \\ \frac{\partial}{\partial t} u(x,0) = -3 (x-5)^2 (x-1) x^2 & 0. \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 0.3 by means of a Fourier series of order 12 .

1) u (1 , 0.3) = 0.455954

2) u (1 , 0.3) = 0.374769

3) u (1 , 0.3) = −5.6753

4) u (1 , 0.3) = −52.6615

5) u (1 , 0.3) = −8.51932

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 7

Exercise 1

Compute $\int_D (x\, y^2)\, dx\, dy\, dz$ for $D = \{x^5 z \le y^3 \le 8 x^5 z,\ 1 \le x^6 y z^3 \le 9,\ 2 z^3 \le x^2 y^4 \le 10 z^3,\ x > 0,\ y > 0,\ z > 0\}$

1) −0.167858

2) 0.332142

3) −1.06786

4) 2.03214

5) −0.0678579

Exercise 2

Compute the mean curvature for X(u,v) =
{v Cos[u], 2 v − 2 v Cos[u] + v Sin[u], −3 v − 2 v Cos[u] − 2 v Sin[u]} at the point (u,v) = (3, −4).

1) H(3, −4) = −5.6481

2) H(3, −4) = 1.84944

3) H(3, −4) = 1.54722

4) H(3, −4) = 2.20163

5) H(3, −4) = 0.00356067

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = -(x-1)^2 \left(x - \frac{4}{5}\right) x^2 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{9}{10}$ and the moment t = 0.8 by means of a Fourier series of order 8.

1) $u\left(\frac{9}{10}, 0.8\right) = 1.59994$

2) $u\left(\frac{9}{10}, 0.8\right) = 0$

3) $u\left(\frac{9}{10}, 0.8\right) = -1.1805$

4) $u\left(\frac{9}{10}, 0.8\right) = 1.54722$

5) $u\left(\frac{9}{10}, 0.8\right) = 2.20163$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 8

Exercise 1

Given the system

$-2\,x\,y\,u_3 - 2\,x\,u_3\,u_4 + y\,u_4\,u_5 == 160$
$2\,u_2\,u_3^2 - 2\,y\,u_5 + 2\,x\,u_1\,u_5 == 50$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-4, -5, -4, -5, -4, 0, 5)$. Compute if possible $\frac{\partial y}{\partial u_5}(-4, -5, -4, 0, 5)$.

1) $\frac{\partial y}{\partial u_5}(-4, -5, -4, 0, 5) = -\frac{21}{11}$

2) $\frac{\partial y}{\partial u_5}(-4, -5, -4, 0, 5) = -\frac{20}{11}$

3) $\frac{\partial y}{\partial u_5}(-4, -5, -4, 0, 5) = -\frac{18}{11}$

4) $\frac{\partial y}{\partial u_5}(-4, -5, -4, 0, 5) = -\frac{19}{11}$

5) $\frac{\partial y}{\partial u_5}(-4, -5, -4, 0, 5) = -\frac{17}{11}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-6\,u^2 + u\,(3 + 6\,v) + 2\,(-9 + v + 12\,v^2),\ -12 - 4\,u^2 + v + 16\,v^2 + u\,(2 + 4\,v),\ -6 - 2\,u^2 - v + 8\,v^2 + 2\,u\,(1 + v)\}$ at the point $(u,v) = (4, 0)$.

1) $K(4, 0) = -5.8479 \times 10^{-6}$

2) $K(4, 0) = 5.31876$

3) $K(4, 0) = -8.82397$

4) $K(4, 0) = -3.60031$

5) $K(4, 0) = -5.63198$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0\le t \\ u(x,0)=-3\,(x-2)\,(x-1)\,x & 0\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{4}{5}$ and the moment $t=0.5$ by means of a Fourier series of order 10 .

1) $u(\frac{4}{5}, 0.5) = -4.16089$

2) $u(\frac{4}{5}, 0.5) = -1.82575\times 10^{-9}$

3) $u(\frac{4}{5}, 0.5) = 5.31876$

4) $u(\frac{4}{5}, 0.5) = 8.56956$

5) $u(\frac{4}{5}, 0.5) = -3.60031$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 9

Exercise 1

Given the function

$f(x, y, z) = 19 - 4x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, -0.972942, -5.04724}

2) We have a maximum at {?, -1.27294, -4.64724}

3) We have a maximum at {-0.160074, -0.872942, ?}

4) We have a maximum at {-0.760074, -0.972942, ?}

Exercise 2

Consider the vectorial field $F(x, y, z) = (3, 2yz^2 e^{yz} + 2z e^{yz} + 6y, 2y^2 z e^{yz} + 2y e^{yz})$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the value of the potential at the point $p = (10, 2, -9)$.

1) $-53 - \frac{36}{e^{18}}$ 2) $46 - \frac{36}{e^{18}}$ 3) $\frac{101}{2} - \frac{36}{e^{18}}$ 4) $163 - \frac{36}{e^{18}}$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{4x}{3} & 0 \le x \le 3 \\ 16 - 4x & 3 \le x \le 4 \end{cases} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 3$ and the moment $t = 0.5$ by means of a Fourier series of order 11.

1) u(3, 0.5) = 3.82346

2) u(3, 0.5) = -1.30294

3) u(3, 0.5) = 2.00455

4) u(3, 0.5) = -2.31844

5) u(3, 0.5) = -1.95861

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 10

Exercise 1

Given the function

$f(x, y, z) = -10 + 4x - x^2 - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-1.325, -4.43512, ?}

2) We have a minimum at {2, ?, 0}

3) We have a minimum at {?, -4.23512, -0.4}

4) We have a minimum at {-0.625, -4.43512, ?}

Exercise 2

Consider the vectorial field $F(x, y, z) = \left(\frac{z\,(y z - 3 x z)}{x z + 1} - 3 z \log(x z + 1), z \log(x z + 1) - 4, \frac{x\,(y z - 3 x z)}{x z + 1} + (y - 3x) \log(x z + 1) \right)$. Compute the potential function for this field whose potential at the origin is 5 .

. Calculate the value of the potential at the point p= (8 , 3 , 8) .

1) $\frac{9891}{5} - 168 \,\text{Log}[65]$ 2) $-2134 - 168 \,\text{Log}[65]$

3) $-\frac{12123}{10} - 168 \,\text{Log}[65]$ 4) $-7 - 168 \,\text{Log}[65]$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{5x}{2} & 0 \le x \le 2 \\ \frac{5x}{\pi-2} - \frac{10}{\pi-2} - 5 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.7 by means of a Fourier series of order 8 .

1) u (2 , 0.7) = -4.51508

2) u (2 , 0.7) = -2.5

3) u (2 , 0.7) = -3.81399

4) u (2 , 0.7) = 1.6034

5) u (2 , 0.7) = 2.45847

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 11

Exercise 1

Given the system

$-u^2 v + 3 u v^2 - 2 v^2 x + x^2 + 2 u y - 3 v^2 y - 2 v x y - 2 y^2 - 3 x y^2 == -166$

$2 v^3 + x - 2 x^2 + u x y == 24$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (2 , 5 , 3 , 0). Compute if possible $\frac{\partial x}{\partial v}(3,0)$.

1) $\frac{\partial x}{\partial v}(3,0) = \frac{88}{83}$

2) $\frac{\partial x}{\partial v}(3,0) = \frac{89}{83}$

3) $\frac{\partial x}{\partial v}(3,0) = \frac{90}{83}$

4) $\frac{\partial x}{\partial v}(3,0) = \frac{87}{83}$

5) $\frac{\partial x}{\partial v}(3,0) = \frac{91}{83}$

Exercise 2

Compute the Gauss curvature for X(u,v) =
{Cos[u] (3 + 2 Cos[v]) - 2 (3 + 2 Cos[v]) Sin[u] - 6 Sin[v],
2 Cos[u] (3 + 2 Cos[v]) + (3 + 2 Cos[v]) Sin[u] + 2 Sin[v],
(3 + 2 Cos[v]) Sin[u] + 3 Sin[v]} at the point (u,v) = (1 , 0).

1) K (1 , 0) = -7.22003

2) K (1 , 0) = 0.750255

3) K (1 , 0) = 7.63462

4) K (1 , 0) = 6.19386

5) K (1 , 0) = 0.0000200658

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} x & 0 \le x \le 1 \\ \frac{x}{2} + \frac{1}{2} & 1 \le x \le 3 \\ -\frac{2x}{\pi-3} + \frac{6}{\pi-3} + 2 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.1$ by means of a Fourier series of order 12 .

1) $u(2, 0.1) = 4.73565$

2) $u(2, 0.1) = 7.63462$

3) $u(2, 0.1) = 1.0486$

4) $u(2, 0.1) = -7.25199$

5) $u(2, 0.1) = -7.22003$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 12

Exercise 1

Compute the volume of the domain limited by the plane $9x + 2z == 1$ and the paraboloid $z == x^2 + y^2$.

1) 48.6026

2) 116.441

3) 169.754

4) 26.7887

5) 20.1656

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] \dashrightarrow R^2$

$$r(t) = \left\{ \frac{\left(\frac{1}{\sqrt{2}} - \frac{\sin(t)}{\sqrt{2}}\right)\cos(t)\,(\cos(t)+5)}{\sin^2(t)+1},\ \frac{\left(\frac{\sin(t)}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\cos(t)\,(\cos(t)+5)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 25.8584 2) 28.3584 3) 10.8584 4) 40.8584

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-3)(x-1)\,x^2\,(x-\pi)^2 & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -\frac{5x}{3} & 0 \le x \le 3 \\ \frac{5x}{\pi-3} - \frac{15}{\pi-3} - 5 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 2$ and the moment $t = 0.6$ by means of a Fourier series of order 8.

1) u(2 , 0.6) = −7.94821

2) u(2 , 0.6) = 1.34138

3) u(2 , 0.6) = −8.00537

4) u(2 , 0.6) = −5.18534

5) u(2 , 0.6) = 3.88453

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 13

Exercise 1

Given the function

$f(x, y, z) = -7 - x^2 + 4y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0.9, 2.3, ?}

2) We have a maximum at {?, 2.3, 4.5}

3) We have a maximum at {?, 2, 3}

4) We have a maximum at {0.3, 2.9, ?}

Exercise 2

Consider the vectorial field $F(x, y, z) = (3y - 2xy^2, -2x^2y + 3x - 6y^2z^3, -6y^3z^2)$. Compute the potential function for this field whose potential at the origin is −5 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) −7.48611 2) −8.98611 3) 8.51389 4) −4.48611

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{4x}{3} & 0 \le x \le 3 \\ -\frac{4x}{\pi-3} + \frac{12}{\pi-3} + 4 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.6 by means of a Fourier series of order 10 .

1) u(1, 0.6) = −4.05853

2) u(1, 0.6) = 2.79393

3) u(1, 0.6) = 1.09568

4) u(1, 0.6) = 1.99994

5) u(1, 0.6) = −3.11651

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 14

Exercise 1

Given the function

$f(x, y, z) = 13 - 4x + x^2 + y^2 - 6z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-2.5937, 0., ?}

2) We have a maximum at {-2.4937, 0.5, ?}

3) We have a maximum at {2, ?, 3}

4) We have a maximum at {?, 0.3, -1.40503}

Exercise 2

Compute the Gauss curvature for X(u,v) = $\{-4 - 14u^2 - 17v + 8v^2 + u(-1 + 10v), -4 - 14u^2 - 17v + 8v^2 + 2u(-1 + 5v), -2 - 7u^2 - 9v + 4v^2 + u(-1 + 5v)\}$ at the point (u,v) = (1, 6).

1) K(1, 6) = -1.69625×10^{-6}

2) K(1, 6) = -6.92336

3) K(1, 6) = 6.79005

4) K(1, 6) = -3.63084

5) K(1, 6) = -8.54855

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 15x & 0\le x\le \frac{2}{5} \\ 10-10x & \frac{2}{5}\le x\le 1 \end{cases} & 0\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{7}{10}$ and the moment $t=0.8$ by means of a Fourier series of order 10 .

1) $u(\frac{7}{10}, 0.8) = 3.94064$

2) $u(\frac{7}{10}, 0.8) = 3.84529$

3) $u(\frac{7}{10}, 0.8) = 1.75432$

4) $u(\frac{7}{10}, 0.8) = 3.$

5) $u(\frac{7}{10}, 0.8) = -4.26626$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 15

Exercise 1

Given the function

$f(x,y,z) = -12 + 2x - x^2 + 6y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-0.872811, -0.783724, ?}

2) We have a minimum at {-0.472811, -1.08372, ?}

3) We have a minimum at {?, -0.383724, -2.71843}

4) We have a minimum at {?, 3, 3}

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{v + \text{Cos}[u] + 2\,\text{Sin}[u], \text{Sin}[u], v + \text{Sin}[u]\}$ at the point $(u,v) = (5, -5)$.

1) K (5 , -5) = 1.71586

2) K (5 , -5) = 3.56379

3) K (5 , -5) = 0

4) K (5 , -5) = 5.36936

5) K (5 , -5) = -7.88584

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -3x & 0 \le x \le 2 \\ 10x - 26 & 2 \le x \le 3 \\ -\frac{4x}{\pi-3} + \frac{12}{\pi-3} + 4 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.5 by means of a Fourier series of order 8 .

1) u(2 , 0.5) = 3.61943

2) u(2 , 0.5) = 3.44594

3) u(2 , 0.5) = -2.13448

4) u(2 , 0.5) = -3.51999

5) u(2 , 0.5) = -1.24328

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 16

Exercise 1

Given the system

$v^2 + 2\,v\,x - 2\,v^2\,x - 2\,v\,x^2 - 2\,u^2\,y - 3\,u\,v\,y + 3\,v\,x\,y == 188$
$3\,v^2 - 2\,v^2\,x + 3\,x^3 - 2\,u\,y + 2\,x\,y - 3\,u\,x\,y - 2\,y^2 + 3\,v\,y^2 == 36$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (-2 , -4 , 3 , 2). Compute if possible $\frac{\partial x}{\partial u}(3,2)$.

1) $\frac{\partial x}{\partial u}(3,2) = \frac{29}{31}$

2) $\frac{\partial x}{\partial u}(3,2) = \frac{27}{31}$

3) $\frac{\partial x}{\partial u}(3,2) = \frac{28}{31}$

4) $\frac{\partial x}{\partial u}(3,2) = \frac{26}{31}$

5) $\frac{\partial x}{\partial u}(3,2) = \frac{30}{31}$

Exercise 2

Compute the Gauss curvature for X(u,v) =
$\{-v + (1 + 3\,v^2)\,\text{Cos}[u] + (1 + 3\,v^2)\,\text{Sin}[u],\ (1 + 3\,v^2)\,\text{Sin}[u],\ (1 + 3\,v^2)\,\text{Cos}[u] - (1 + 3\,v^2)\,\text{Sin}[u]\}$
at the point (u,v) = (5 , 0).

1) K (5 , 0) = -0.652683

2) K (5 , 0) = -5.83126

3) K (5 , 0) = 7.49039

4) K (5 , 0) = -20.8499

5) K (5 , 0) = 1.93326

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = 3\,(x-4)^2\,(x-3)\,(x-2)\,x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 3$
and the moment $t = 0.6$ by means of a Fourier series of order 10 .

1) $u(3, 0.6) = -7.90094$

2) $u(3, 0.6) = 3.71184$

3) $u(3, 0.6) = -0.652683$

4) $u(3, 0.6) = -5.71776$

5) $u(3, 0.6) = -3.54642$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 17

Exercise 1

Given the system

$-x^3 - 2 u_1^3 + 3 y u_3 == -70$

$-3 x u_1^2 - 2 x u_4 - 2 y u_1 u_4 == -22$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (2, -4, 1, -2, 5, -4)$. Compute if possible $\frac{\partial y}{\partial u_2}(1, -2, 5, -4)$.

1) $\frac{\partial y}{\partial u_2}(1, -2, 5, -4) = 2$

2) $\frac{\partial y}{\partial u_2}(1, -2, 5, -4) = 1$

3) $\frac{\partial y}{\partial u_2}(1, -2, 5, -4) = 0$

4) $\frac{\partial y}{\partial u_2}(1, -2, 5, -4) = 4$

5) $\frac{\partial y}{\partial u_2}(1, -2, 5, -4) = 3$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-v + 4(1+v^2)\,\mathrm{Cos}[u] - (1+v^2)\,\mathrm{Sin}[u],\ -3(1+v^2)\,\mathrm{Cos}[u] + (1+v^2)\,\mathrm{Sin}[u],\ v - 3(1+v^2)\,\mathrm{Cos}[u] + (1+v^2)\,\mathrm{Sin}[u]\}$ at the point $(u,v) = (1, -4)$.

1) $K(1, -4) = 1.43599$

2) $K(1, -4) = 1.27544$

3) $K(1, -4) = -8.5574$

4) $K(1, -4) = -3.51051 \times 10^{-6}$

5) $K(1, -4) = -2.71241$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0 \le t \\ u(x,0)=3\,(x-1)^2\left(x-\frac{1}{2}\right)x^2 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{1}{10}$ and the moment $t=0.5$ by means of a Fourier series of order 9.

1) $u\left(\frac{1}{10}, 0.5\right) = 7.9827$

2) $u\left(\frac{1}{10}, 0.5\right) = -2.71241$

3) $u\left(\frac{1}{10}, 0.5\right) = 7.27831$

4) $u\left(\frac{1}{10}, 0.5\right) = -3.62895$

5) $u\left(\frac{1}{10}, 0.5\right) = 0$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 18

Exercise 1

Compute $\int_D (y^2)\,dx\,dy\,dz$ for D=

$\{9x^4y^2 \le z^2 \le 12x^4y^2,\ 4 \le x^2y^7z^5 \le 6,\ 9x^5y^3 \le z^4 \le 14x^5y^3,\ x>0,\ y>0,\ z>0\}$

1) 2.00218

2) 1.80218

3) 0.802179

4) 0.00217899

5) −0.397821

Exercise 2

Compute the area of the domain whose boundary is the curve

$r:[0,\pi] \longrightarrow R^2$

$r(t)=\left\{\sin(2t)\,(6\cos(t)+7)\left(-\frac{\sin(t)}{\sqrt{2}}-\frac{\cos(t)}{\sqrt{2}}\right),\ \sin(2t)\,(6\cos(t)+7)\left(\frac{\cos(t)}{\sqrt{2}}-\frac{\sin(t)}{\sqrt{2}}\right)\right\}$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 52.6217 2) 11.0217 3) 31.8217 4) 68.2217

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 4x & 0\le x\le 1 \\ 6-2x & 1\le x\le 3 \end{cases} & 0\le x\le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
and the moment t= 0.9 by means of a Fourier series of order 9 .

1) u(1 , 0.9) = 6.60316

2) u(1 , 0.9) = 0

3) u(1 , 0.9) = −4.57997

4) u(1 , 0.9) = −4.11992

5) u(1 , 0.9) = −6.2348

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 19

Exercise 1

Compute the volume of the domain limited by the plane $10x + z == 8$ and the paraboloid $z == 8x^2 + 8y^2$.

1) 16.8788

2) 5.60228

3) 63.7568

4) 24.3013

5) 120.551

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] \longrightarrow R^2$

$$r(t) = \left\{ \frac{\left(-\frac{1}{2}\sqrt{3}\sin(t) - \frac{1}{2}\right)\cos(t)\,(5\cos(t)+5)}{\sin^2(t)+1}, \frac{\left(\frac{\sqrt{3}}{2} - \frac{\sin(t)}{2}\right)\cos(t)\,(5\cos(t)+5)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 46.4602 2) 18.8602 3) 74.0602 4) 37.2602

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{x}{2} & 0 \le x \le 2 \\ \frac{x}{\pi-2} - \frac{2}{\pi-2} - 1 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -3x & 0 \le x \le 3 \\ \frac{9x}{\pi-3} - \frac{27}{\pi-3} - 9 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x = 1 and the moment t = 0.5 by means of a Fourier series of order 12.

1) u(1 , 0.5) = −0.288577

2) u(1 , 0.5) = 3.97012

3) u(1 , 0.5) = 5.83138

4) u(1 , 0.5) = 0.915543

5) u(1 , 0.5) = −6.2784

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 20

Exercise 1

Compute $\int_D (2y + y^3)\,dx\,dy\,dz$ for D=

$\{8z^2 \leq x^5 y^2 \leq 14z^2,\ 8x^2 z^5 \leq y^6 \leq 17x^2 z^5,\ 6x^4 y z^3 \leq 1 \leq 12x^4 y z^3,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.501582

2) −0.798418

3) −1.19842

4) 0.0015819

5) −0.398418

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0,\pi] \dashrightarrow R^2$

$r(t) = \left\{\sin(2t)\,(5\cos(t)+6)\left(-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\cos(t)}{2\sqrt{2}}\right),\ \sin(2t)\,(5\cos(t)+6)\left(\frac{(1+\sqrt{3})\cos(t)}{2\sqrt{2}} - \frac{(\sqrt{3}\right.\right.$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 60.8918 2) 15.2918 3) 64.6918 4) 38.0918

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \leq t \\ u(x,0) = \begin{cases} -2x & 0 \leq x \leq 2 \\ 8x - 20 & 2 \leq x \leq 3 \\ -\frac{4x}{\pi-3} + \frac{12}{\pi-3} + 4 & 3 \leq x \leq \pi \end{cases} & 0 \leq x \leq \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.9 by means of a Fourier series of order 12 .

1) u(2 , 0.9) = −1.1831

2) u(2 , 0.9) = −2.64319

3) u(2 , 0.9) = −0.0374311

4) u(2 , 0.9) = 0.662322

5) u(2 , 0.9) = 3.24279

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 21

Exercise 1

Given the system

$-v\,x + 3\,y^3 == -370$
$2\,w + 3\,u\,w\,x + x\,y == 22$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (-1 , -5 , -5 , 5 , 1). Compute if possible $\frac{\partial x}{\partial u}$ (-5 , 5 , 1) .

1) $\frac{\partial x}{\partial u}(-5, 5, 1) = -\frac{135}{901}$

2) $\frac{\partial x}{\partial u}(-5, 5, 1) = -\frac{133}{901}$

3) $\frac{\partial x}{\partial u}(-5, 5, 1) = -\frac{132}{901}$

4) $\frac{\partial x}{\partial u}(-5, 5, 1) = -\frac{131}{901}$

5) $\frac{\partial x}{\partial u}(-5, 5, 1) = -\frac{134}{901}$

Exercise 2

Compute the Gauss curvature for X(u,v) = {v - 3 Cos[u] + 2 Sin[u], -2 Cos[u] + Sin[u], v} at the point (u,v) = (2 , 6) .

1) K (2 , 6) = 1.569

2) K (2 , 6) = 8.10906

3) K (2 , 6) = -7.59956

4) K (2 , 6) = 0

5) K (2 , 6) = 0.660933

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-5)^2\,(x-4)\,x & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.5$ by means of a Fourier series of order 8 .

1) u (1 , 0.5) = −0.313143

2) u (1 , 0.5) = −1.47748

3) u (1 , 0.5) = 52.1273

4) u (1 , 0.5) = 3.16789

5) u (1 , 0.5) = 5.5113

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 22

Exercise 1

Given the function

$f(x, y, z) = 15 - 2x + x^2 - 4y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {1.16426, ?, -0.945701}

2) We have a minimum at {0.0294156, ?, 0.756561}

3) We have a minimum at {1, ?, 0}

4) We have a minimum at {?, 1.8914, 0.}

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{\text{Cos}[u], v + \text{Sin}[u], v\}$ at the point $(u,v) = (2, 10)$.

1) K (2 , 10) = -8.38201

2) K (2 , 10) = 1.57496

3) K (2 , 10) = -6.57615

4) K (2 , 10) = 4.36051

5) K (2 , 10) = 0

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = 3\,(x-3)\,(x-1)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.2 by means of a Fourier series of order 10 .

1) u (2 , 0.2) = -4.65667

2) u (2 , 0.2) = -0.493843

3) u (2 , 0.2) = -1.00438

4) u (2 , 0.2) = 0.569273

5) u (2 , 0.2) = 4.91748

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 23

Exercise 1

Given the system

$3\,y\,u_5 - x\,u_1\,u_5 == 21$
$-2\,x\,y\,u_1 + 2\,u_1^3 - 3\,y\,u_4 == -24$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (1, -2, 1, 4, 1, -5, -3)$. Compute if possible $\frac{\partial x}{\partial u_1}(1, 4, 1, -5, -3)$.

1) $\frac{\partial x}{\partial u_1}(1, 4, 1, -5, -3) = -\frac{42}{25}$

2) $\frac{\partial x}{\partial u_1}(1, 4, 1, -5, -3) = -\frac{39}{25}$

3) $\frac{\partial x}{\partial u_1}(1, 4, 1, -5, -3) = -\frac{41}{25}$

4) $\frac{\partial x}{\partial u_1}(1, 4, 1, -5, -3) = -\frac{43}{25}$

5) $\frac{\partial x}{\partial u_1}(1, 4, 1, -5, -3) = -\frac{8}{5}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{u,\ 14 - 6\,u^2 + u\,(2 - 8\,v) + 17\,v + 4\,v^2,\ 7 - 3\,u^2 + u\,(2 - 4\,v) + 9\,v + 2\,v^2\}$ at the point $(u,v) = (-1, -1)$.

1) $K(-1, -1) = -5.51413$

2) $K(-1, -1) = -8.79887$

3) $K(-1, -1) = 1.77531$

4) $K(-1, -1) = -0.000242665$

5) $K(-1, -1) = -5.21191$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -7x & 0\le x\le 1 \\ 7x-14 & 1\le x\le 2 \end{cases} & 0\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{11}{10}$ and the moment $t=0.9$ by means of a Fourier series of order 11 .

1) $u(\frac{11}{10}, 0.9) = 6.9729$

2) $u(\frac{11}{10}, 0.9) = -2.87067$

3) $u(\frac{11}{10}, 0.9) = 3.08632$

4) $u(\frac{11}{10}, 0.9) = -3.35453$

5) $u(\frac{11}{10}, 0.9) = -0.000777723$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 24

Exercise 1

Compute $\int_D (y+2z)\,dx\,dy\,dz$ for D=
$\{9x^5y^3 \le 1 \le 15x^5y^3,\ 8y^3 \le x^4z^6 \le 13y^3,\ 3x^6 \le y^2z^7 \le 9x^6,\ x>0,\ y>0,\ z>0\}$

1) 0.201505

2) 0.101505

3) 0.00150543

4) -1.69849

5) 1.10151

Exercise 2

Compute the mean curvature for X(u,v) =
{Cos[u] (3 + Cos[v]) - 3 Sin[v], (3 + Cos[v]) Sin[u] - 3 Sin[v], -2 (3 + Cos[v]) Sin[u] + 7 Sin[v]}
at the point (u,v) = (3 , 3) .

1) H (3 , 3) = -1.64586

2) H (3 , 3) = -5.44729

3) H (3 , 3) = -0.0666312

4) H (3 , 3) = 5.75325

5) H (3 , 3) = -2.46754

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = 2(x-4)(x-1)x^2 & 0 \le x \le 4 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} 4x & 0 \le x \le 1 \\ \frac{16}{3} - \frac{4x}{3} & 1 \le x \le 4 \end{cases} & 0. \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 0.4 by means of a Fourier series of order 11 .

1) u (1 , 0.4) = -2.46754

2) u (1 , 0.4) = -5.19955

3) u (1 , 0.4) = -5.12193

4) u (1 , 0.4) = -9.63589

5) u (1 , 0.4) = -7.23999

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 25

Exercise 1

Given the system

$-x^2 y - y u_1 == -4$

$x y^2 - 3 x u_1 + 2 y^2 u_3 + 3 u_3 u_4 == 22$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (1, -4, -2, -5, 0, 0)$. Compute if possible $\frac{\partial y}{\partial u_1}(-2, -5, 0, 0)$.

1) $\frac{\partial y}{\partial u_1}(-2, -5, 0, 0) = -\frac{56}{43}$

2) $\frac{\partial y}{\partial u_1}(-2, -5, 0, 0) = -\frac{55}{43}$

3) $\frac{\partial y}{\partial u_1}(-2, -5, 0, 0) = -\frac{52}{43}$

4) $\frac{\partial y}{\partial u_1}(-2, -5, 0, 0) = -\frac{53}{43}$

5) $\frac{\partial y}{\partial u_1}(-2, -5, 0, 0) = -\frac{54}{43}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{3 v + 5 (1 + 2 v^2) \text{Cos}[u] + 2 (1 + 2 v^2) \text{Sin}[u], v + 2 (1 + 2 v^2) \text{Cos}[u] + (1 + 2 v^2) \text{Sin}[u], 4 v + 6 (1 + 2 v^2) \text{Cos}[u] + 3 (1 + 2 v^2) \text{Sin}[u]\}$ at the point $(u,v) = (5, -1)$.

1) $K(5, -1) = -0.0000338933$

2) $K(5, -1) = -1.63962$

3) $K(5, -1) = 3.09094$

4) $K(5, -1) = 3.28585$

5) $K(5, -1) = 3.76679$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t)=u(4,t)=0 & 0 \le t \\ u(x,0) = (x-4)^2\ (x-2)\ (x-1)\ x^2 & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 1. by means of a Fourier series of order 11 .

1) u(2 , 1.) = −8.80329

2) u(2 , 1.) = 4.12155

3) u(2 , 1.) = 1.17668

4) u(2 , 1.) = −1.63962

5) u(2 , 1.) = 3.27031

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 26

Exercise 1

Given the system

$-2+2u^2+u^3-x+u^2x-3ux^2-3x^3-3u^2y-2uxy-2xy^2==-24$

$-2+3u^3+2x-2u^2x+3ux^2+2y-uy+u^2y+3uy^2==-6$

determine if it is possible to solve for variables x , y in terms of variable u arround the point p=(x,y,u)=(1,-3,0). Compute if possible $\frac{\partial y}{\partial u}(0)$.

1) $\frac{\partial y}{\partial u}(0)=-\frac{93}{8}$

2) $\frac{\partial y}{\partial u}(0)=-\frac{23}{2}$

3) $\frac{\partial y}{\partial u}(0)=-\frac{91}{8}$

4) $\frac{\partial y}{\partial u}(0)=-\frac{45}{4}$

5) $\frac{\partial y}{\partial u}(0)=-\frac{89}{8}$

Exercise 2

Compute the Gauss curvature for X(u,v) = {Cos[u], 3 v + 2 Cos[u] + 4 Sin[u], v + Sin[u]} at the point (u,v) = (2, -10).

1) K(2, -10) = 8.88674

2) K(2, -10) = -6.41826

3) K(2, -10) = 7.9715

4) K(2, -10) = 4.88353

5) K(2, -10) = 0

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{5x}{2} & 0 \le x \le 2 \\ 9 - 2x & 2 \le x \le 3 \\ -\frac{3x}{\pi-3} + \frac{9}{\pi-3} + 3 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.1$ by means of a Fourier series of order 12 .

1) $u(1, 0.1) = -7.35251$

2) $u(1, 0.1) = 2.49112$

3) $u(1, 0.1) = -0.0790297$

4) $u(1, 0.1) = -0.669675$

5) $u(1, 0.1) = 3.38495$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 27

Exercise 1

Given the function

$f(x,y,z) = 7 + x^2 - 2y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{4} \leq 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, -4., 0.}

2) We have a maximum at {?, 1, 0}

3) We have a maximum at {?, -3.9, 0.4}

4) We have a maximum at {-0.4, -4.4, ?}

Exercise 2

Consider the vectorial field $F(x,y,z) = (3x^2y^3(1-3yz) + 2x, -3x^3y^3z + 3x^3y^2(1-3yz) - 3, -3x^3y^4)$. Compute the potential function for this field whose potential at the origin is 3.

. Calculate the value of the potential at the point $p = (-1, -5, -5)$.

1) $\frac{249237}{10}$ 2) $-\frac{27693}{2}$ 3) -9231 4) $-\frac{27693}{10}$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = 2(x-2)x(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.2$ by means of a Fourier series of order 8.

1) $u(2, 0.2) = 0.528318$

2) $u(2, 0.2) = 1.36601$

3) $u(2, 0.2) = 3.11696$

4) $u(2, 0.2) = -2.32944$

5) $u(2, 0.2) = -3.72383$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 28

Exercise 1

Compute $\int_D (y^5)\,dx\,dy\,dz$ for $D=\{8 z^4 \le x^4 y^5 \le 17 z^4,\ 9 y^9 \le x^5 z^4 \le 17 y^9,\ 7 x \le z^9 \le 9 x,\ x>0,\ y>0,\ z>0\}$

1) 1.70215

2) -1.99785

3) 0.90215

4) -1.09785

5) 0.00215015

Exercise 2

Consider the vectorial field $F(x,y,z)=(2 y z e^{x y z} + y z e^{x y z}(2 x y z - 2 y z),\ (2 x z - 2 z) e^{x y z} + x z e^{x y z}(2 x y z - 2 y z),\ (2 x y - 2 y) e^{x y z} + x y e^{x y z}(2 x y z - 2 y z))$. Compute the potential function for this field whose potential at the origin is -2.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) 1.607 2) 1.007 3) -11.293 4) -2.293

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0 \le t \\ u(x,0) = -3 (x-2)^2 (x-1) x^2 & 0 \le x \le 2 \\ \frac{\partial}{\partial t} u(x,0) = \begin{cases} 6x & 0 \le x \le 1 \\ 12 - 6x & 1 \le x \le 2 \end{cases} & 0. \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=\frac{7}{10}$

and the moment $t=0.8$ by means of a Fourier series of order 8.

1) $u(\frac{7}{10}, 0.8) = 8.09111$

2) $u(\frac{7}{10}, 0.8) = 0.212014$

3) $u(\frac{7}{10}, 0.8) = 0.362417$

4) $u(\frac{7}{10}, 0.8) = -1.13754$

5) $u(\frac{7}{10}, 0.8) = 1.84553$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 29

Exercise 1

Given the system

$2u + 2u^2 + 2u^3 + 3u^2 v - 3v^3 - ux - 3vx + 2vy - 3uvy == 128$

$2x^2 + 2x^2 y + 2y^2 == 42$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (1 , 4 , 2 , -3). Compute if possible $\frac{\partial y}{\partial u}(2, -3)$.

1) $\frac{\partial y}{\partial u}(2, -3) = -\frac{106}{19}$

2) $\frac{\partial y}{\partial u}(2, -3) = -\frac{108}{19}$

3) $\frac{\partial y}{\partial u}(2, -3) = -\frac{107}{19}$

4) $\frac{\partial y}{\partial u}(2, -3) = -\frac{110}{19}$

5) $\frac{\partial y}{\partial u}(2, -3) = -\frac{109}{19}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-v + (1 + v^2)\,\mathrm{Cos}[u],\ 2v - (1 + v^2)\,\mathrm{Cos}[u] + (1 + v^2)\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (2, -4)$.

1) K (2 , -4) = 0.816152

2) K (2 , -4) = 8.68152

3) K (2 , -4) = -0.0000565296

4) K (2 , -4) = 8.98777

5) K (2 , -4) = 2.48859

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t)=u(3,t)=0 & 0 \le t \\ u(x,0) = \begin{cases} -5x & 0 \le x \le 1 \\ 7x-12 & 1 \le x \le 2 \\ 6-2x & 2 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
 and the moment t= 0.9 by means of a Fourier series of order 9 .

1) u (1 , 0.9) = 8.98777

2) u (1 , 0.9) = 1.25357

3) u (1 , 0.9) = -0.555993

4) u (1 , 0.9) = 2.0754

5) u (1 , 0.9) = -6.40604

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 30

Exercise 1

Given the function

$f(x, y, z) = 4 - 4x + x^2 + y^2 - 6z + z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {2, ?, 3}

2) We have a minimum at {2.3, 0.6, ?}

3) We have a minimum at {3.5, -0.9, ?}

4) We have a minimum at {1.4, ?, 3.9}

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-v + (1 + 2v^2)\,\mathrm{Cos}[u] + 2(1 + 2v^2)\,\mathrm{Sin}[u],\ (1 + 2v^2)\,\mathrm{Sin}[u],\ 3v - 2(1 + 2v^2)\,\mathrm{Cos}[u] - 5(1 + 2v^2)\,\mathrm{Sin}[u]\}$ at the point $(u,v) = (4, -4)$.

1) $K(4, -4) = 3.78081$

2) $K(4, -4) = 6.94862$

3) $K(4, -4) = 7.88244$

4) $K(4, -4) = -8.42058$

5) $K(4, -4) = -6.26849 \times 10^{-8}$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -5x & 0 \le x \le 1 \\ \frac{5x}{3} - \frac{20}{3} & 1 \le x \le 4 \end{cases} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.5$ by means of a Fourier series of order 9.

1) $u(2, 0.5) = -2.50002$

2) $u(2, 0.5) = -0.944879$

3) $u(2, 0.5) = 3.86034$

4) $u(2, 0.5) = -0.263169$

5) $u(2, 0.5) = 2.10045$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 31

Exercise 1

Given the system

$2\, x\, y\, u_4 == -30$
$-3\, x^2\, y + 2\, u_4^2 - 3\, u_2\, u_5^2 == -42$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-5, 1, 5, -5, 1, 3, -1)$. Compute if possible $\frac{\partial x}{\partial u_3}(5, -5, 1, 3, -1)$.

1) $\frac{\partial x}{\partial u_3}(5, -5, 1, 3, -1) = 0$

2) $\frac{\partial x}{\partial u_3}(5, -5, 1, 3, -1) = 2$

3) $\frac{\partial x}{\partial u_3}(5, -5, 1, 3, -1) = 3$

4) $\frac{\partial x}{\partial u_3}(5, -5, 1, 3, -1) = 4$

5) $\frac{\partial x}{\partial u_3}(5, -5, 1, 3, -1) = 1$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{v + \mathrm{Cos}[u], 4\, v + \mathrm{Sin}[u], v\}$ at the point $(u,v) = (0, 7)$.

1) $K(0, 7) = 7.32784$

2) $K(0, 7) = -1.8805$

3) $K(0, 7) = 0$

4) $K(0, 7) = -3.75119$

5) $K(0, 7) = 3.18353$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-1)^2 \left(x - \frac{7}{10}\right)\left(x - \frac{1}{2}\right) x^2 & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{3}{10}$ and the moment $t = 0.3$ by means of a Fourier series of order 9 .

1) $u\left(\frac{3}{10}, 0.3\right) = -2.44845$

2) $u\left(\frac{3}{10}, 0.3\right) = -7.55608$

3) $u\left(\frac{3}{10}, 0.3\right) = -1.73148 \times 10^{-8}$

4) $u\left(\frac{3}{10}, 0.3\right) = -8.69875$

5) $u\left(\frac{3}{10}, 0.3\right) = -3.75119$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 32

Exercise 1

Compute $\int_D (3 z^4)\, dx dy dz$ for D=

$\{7 x^4 y^9 z^4 \leq 1 \leq 16 x^4 y^9 z^4,\ 2 x^3 \leq y^8 z^7 \leq 8 x^3,\ 6 x^9 y^3 \leq 1 \leq 12 x^9 y^3,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.1536

2) -0.769068

3) 1.92267

4) -0.961335

5) 4.80668

Exercise 2

Consider the vectorial field F (x , y , z) = (

$-y\,(x y + y z)\sin(x y) + y\cos(x y) + 6 x,\ (x + z)\cos(x y) - x\,(x y + y z)\sin(x y),\ y\cos(x y)$

). Compute the potential function for this field whose potential at the origin is 1 .

. Calculate the value of the potential at the point p= (7 , 6 , -10) .

1) 272 - 18 Cos[42]   2) 24 - 18 Cos[42]   3) -100 - 18 Cos[42]   4) 148 - 18 Cos[42]

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \leq t \\ u(x,0) = 3\,(x-5)\,(x-2)\,x & 0 \leq x \leq 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.6 by means of a Fourier series of order 9 .

1) u (2 , 0.6) = -0.207935

2) u (2 , 0.6) = 3.92266

3) u (2 , 0.6) = -6.22511

4) u (2 , 0.6) = 8.78272

5) u (2 , 0.6) = 1.07306

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 33

Exercise 1

Compute $\int_D (z^6)\,dx\,dy\,dz$ for $D=\{5y \le x^5 z^6 \le 13y,\ 2z^4 \le x^2 y^2 \le 3z^4,\ 9x^2 \le z^2 \le 12x^2,\ x>0,\ y>0,\ z>0\}$

1) 25.2605

2) 50.5211

3) −7.57816

4) 20.2084

5) 50.5211

Exercise 2

Consider the vector field $F(x,y,z)=$
$\{-4x+\mathrm{Cos}[2y^2+z^2],\ e^{2x^2+z^2}+5y+4xy,\ -9z-\mathrm{Sin}[2x^2+y^2]\}$ and the surface

$$S\equiv\left(\frac{-8+x}{6}\right)^2+\left(\frac{-2+y}{8}\right)^2+\left(\frac{-5+z}{2}\right)^2=1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 38601.   2) 9650.97   3) −6754.03   4) −21229.

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 2x & 0\le x\le 3 \\ -\frac{6x}{\pi-3}+\frac{18}{\pi-3}+6 & 3\le x\le\pi \end{cases} & 0\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.4$ by means of a Fourier series of order 10 .

1) u(2 , 0.4) = −1.18078

2) u(2 , 0.4) = −0.0544341

3) u(2 , 0.4) = 3.1905

4) u(2 , 0.4) = −3.02482

5) u(2 , 0.4) = 4.70288

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 34

Exercise 1

Compute the volume of the domain limited by the plane $x + 3z == 6$ and the paraboloid $z == 3x^2 + 3y^2$.

1) 2.11383

2) 2.91291

3) 2.90925

4) 9.15916

5) 0.486798

Exercise 2

Consider the vector field $F(x,y,z) = \{4yz + \mathrm{Sin}[y^2 - 2z^2],\ e^{x^2} - 5y,\ e^{x^2-2y^2} + 5x - 3xy\}$ and the surface

$$S \equiv \left(\frac{1+x}{3}\right)^2 + \left(\frac{8+y}{3}\right)^2 + \left(\frac{4+z}{3}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -2772.89 2) -1018.29 3) -2433.29 4) -565.487

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 2(x-2)(x-1)x(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -2x & 0 \le x \le 3 \\ \frac{6x}{\pi-3} - \frac{18}{\pi-3} - 6 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$ and the moment $t = 0.5$ by means of a Fourier series of order 9 .

1) u(1 , 0.5) = 0.345009

2) u(1 , 0.5) = -1.39201

3) u(1 , 0.5) = 8.95929

4) u(1 , 0.5) = 3.65618

5) u(1 , 0.5) = 4.82183

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 35

Exercise 1

Compute the volume of the domain limited by the plane $9x + 7z == 6$ and the paraboloid $z == 4x^2 + 4y^2$.

1) 1.13323

2) 1.12577

3) 0.204618

4) 0.362258

5) 0.176827

Exercise 2

Compute the area of the domain whose boundary is the curve

$r : [0, 2\pi] \dashrightarrow R^2$

$$r(t) = \left\{ \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(2\cos(t)+9)}{\sin^2(t)+1},\ \frac{\left(\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos(t)\,(2\cos(t)+9)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 118.034 2) 84.4336 3) 50.8336 4) 126.434

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = 2(x-2)^2(x-1)x^2 & 0 \le x \le 2 \\ \frac{\partial}{\partial t}u(x,0) = (x-2)(x-1)x^2 & 0. \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{1}{10}$ and the moment $t = 0.7$ by means of a Fourier series of order 12.

1) $u\left(\frac{1}{10}, 0.7\right) = -5.68558$

2) $u\left(\frac{1}{10}, 0.7\right) = -5.84817$

3) $u\left(\frac{1}{10}, 0.7\right) = -8.64182$

4) $u\left(\frac{1}{10}, 0.7\right) = -0.0411625$

5) $u\left(\frac{1}{10}, 0.7\right) = -1.97849$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 36

Exercise 1

Compute $\int_D (y\,z)\,dx\,dy\,dz$ for D=

$\{2\,y\,z^3 \le x^3 \le 8\,y\,z^3,\ 3\,x^9 \le y^7\,z^9 \le 11\,x^9,\ 2\,x^5\,y^2 \le z^7 \le 9\,x^5\,y^2,\ x > 0,\ y > 0,\ z > 0\}$

1) 2.90715×10^8

2) 7.26788×10^8

3) 7.5586×10^8

4) 5.52359×10^8

5) 3.7793×10^8

Exercise 2

Compute the mean curvature for X(u,v) =
{v Cos[u], -v - 2 v Cos[u] + 2 v Sin[u], v - 2 v Cos[u] - v Sin[u]} at the point (u,v) = (6 , 10).

1) H (6 , 10) = 8.66708

2) H (6 , 10) = -6.0537

3) H (6 , 10) = 3.39543

4) H (6 , 10) = -7.02895

5) H (6 , 10) = 0.00224457

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-1)\,x\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
and the moment t= 0.2 by means of a Fourier series of order 9 .

1) u(1 , 0.2) = 3.39543

2) u(1 , 0.2) = -1.49276

3) u(1 , 0.2) = -5.45221

4) u(1 , 0.2) = -6.0537

5) u(1 , 0.2) = -0.201158

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 37

Exercise 1

Compute $\int_D (3\,x\,y^3)\,dx\,dy\,dz$ for D=
$\{3\,x^8 \le y^5\,z^6 \le 8\,x^8,\ 2\,x^4 \le y^4 \le 9\,x^4,\ 7\,y\,z^8 \le x^8 \le 13\,y\,z^8,\ x > 0,\ y > 0,\ z > 0\}$

1) 28.8009

2) 86.4027

3) 57.6018

4) 74.8823

5) 23.0407

Exercise 2

Compute the mean curvature for X(u,v) = {Cos[u] (3 + 2 Cos[v]) + (3 + 2 Cos[v]) Sin[u] + Sin[v], -2 (3 + 2 Cos[v]) Sin[u] - 3 Sin[v], (3 + 2 Cos[v]) Sin[u] + Sin[v]} at the point (u,v) = (1 , 1).

1) H (1 , 1) = -3.12538

2) H (1 , 1) = 0.10636

3) H (1 , 1) = -7.87666

4) H (1 , 1) = 4.83323

5) H (1 , 1) = -5.26533

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{45x}{4} & 0 \le x \le \frac{4}{5} \\ 45x-45 & \frac{4}{5} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ \frac{\partial}{\partial t}u(x,0) = 2(x-1)\left(x-\frac{3}{5}\right)\left(x-\frac{2}{5}\right)x^2 & 0. \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{7}{10}$ and the moment $t = 0.4$ by means of a Fourier series of order 10 .

1) $u(\frac{7}{10}, 0.4) = -7.87666$

2) $u(\frac{7}{10}, 0.4) = -5.26533$

3) $u(\frac{7}{10}, 0.4) = 8.76168$

4) $u(\frac{7}{10}, 0.4) = -1.83437$

5) $u(\frac{7}{10}, 0.4) = 3.37995$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 38

Exercise 1

Given the function

$f(x,y,z) = 24 - 2x + x^2 - 6y + y^2 - 6z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.657104, ?, 1.37987}

2) We have a minimum at {0.795091, ?, 1.1039}

3) We have a minimum at {?, 3, 3}

4) We have a minimum at {1.07107, ?, 1.24189}

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-6v - v\,\mathrm{Cos}[u] + 4v\,\mathrm{Sin}[u],\ -v + v\,\mathrm{Sin}[u],\ -9v - v\,\mathrm{Cos}[u] + 6v\,\mathrm{Sin}[u]\}$ at the point $(u,v) = (1, -4)$.

1) K (1 , -4) = 2.41183

2) K (1 , -4) = 0

3) K (1 , -4) = 4.06713

4) K (1 , -4) = 4.76888

5) K (1 , -4) = -7.4613

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{5x}{2} & 0 \le x \le 2 \\ 7x - 19 & 2 \le x \le 3 \\ -\frac{2x}{\pi-3} + \frac{6}{\pi-3} + 2 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.1$ by means of a Fourier series of order 9 .

1) u(2 , 0.1) = 4.06908

2) u(2 , 0.1) = 2.64938

3) u(2 , 0.1) = -2.03034

4) u(2 , 0.1) = 0.150739

5) u(2 , 0.1) = 2.25952

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 39

Exercise 1

Compute $\int_D (3x + y^2)\,dx\,dy\,dz$ for D=
$\{8x^5y^9 \le z^9 \le 9x^5y^9,\ 8z^9 \le xy^4 \le 12z^9,\ 9x^4z^4 \le y^3 \le 12x^4z^4,\ x>0,\ y>0,\ z>0\}$

1) 1.00003

2) 0.0000282817

3) 1.20003

4) 0.400028

5) -1.99997

Exercise 2

Compute the mean curvature for X(u,v) =
{-2 v + v Cos[u] + 6 v Sin[u], v - v Sin[u], v - 2 v Sin[u]} at the point (u,v) = (0, 8).

1) H(0,8) = -6.75444

2) H(0,8) = -7.56176

3) H(0,8) = -0.888544

4) H(0,8) = 0.000688854

5) H(0,8) = -5.36387

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 7x & 0 \le x \le 1 \\ 13 - 6x & 1 \le x \le 2 \\ -\frac{x}{\pi-2} + \frac{2}{\pi-2} + 1 & 2 \le x \le \pi \\ 0 & \text{True} \end{cases} & 0 \le x \le \pi \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.3 by means of a Fourier series of order 8.

1) u(2, 0.3) = -0.888544

2) u(2, 0.3) = -5.36387

3) u(2, 0.3) = 2.11445

4) u(2, 0.3) = -6.75444

5) u(2, 0.3) = -7.56176

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 40

Exercise 1

Given the function

$f(x,y,z) = -16 + 6x - x^2 + 4y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 2.41619, 0.94235}

2) We have a maximum at {3, 2, ?}

3) We have a maximum at {1.17794, ?, -0.235587}

4) We have a maximum at {2.82705, 0.060316, ?}

Exercise 2

Compute the Gauss curvature for X(u,v) = {v Cos[u] + 2 v Sin[u], -3 v Cos[u] - 5 v Sin[u], v + v Cos[u] + 2 v Sin[u]} at the point (u,v) = (0, 2).

1) K(0,2) = -3.59475

2) K(0,2) = 0.814839

3) K(0,2) = -8.19113

4) K(0,2) = 5.82535

5) K(0,2) = 0

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = -(x-4)^2 (x-3)(x-1) x^2 & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.9 by means of a Fourier series of order 11.

1) u(2, 0.9) = -4.78416

2) u(2, 0.9) = -1.90893

3) u(2, 0.9) = 2.55231

4) u(2, 0.9) = 3.65714

5) u(2, 0.9) = 0.124345

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 41

Exercise 1

Given the function

$f(x, y, z) = 17 - 2x + x^2 - 6y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{4} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {0.7, 2.1, ?}

2) We have a minimum at {?, 1.5, -1.5}

3) We have a minimum at {0.4, ?, 0.9}

4) We have a minimum at {1, 3, ?}

Exercise 2

Consider the vectorial field $F(x, y, z) = \left(-2xy + \frac{xy}{xy+1} + \log(xy+1), \frac{x^2}{xy+1} - x^2 + 6y, 0\right)$. Compute the potential function for this field whose potential at the origin is -3. . Calculate the value of the potential at the point $p = (9, 4, 5)$.

1) $-32 + 9\,\text{Log}[37]$ 2) $-279 + 9\,\text{Log}[37]$ 3) $462 + 9\,\text{Log}[37]$ 4) $\frac{828}{5} + 9\,\text{Log}[37]$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-2)(x-1)x^2(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.8$ by means of a Fourier series of order 9 .

1) $u(1, 0.8) = -0.757003$

2) $u(1, 0.8) = 0.329989$

3) $u(1, 0.8) = -1.30824$

4) $u(1, 0.8) = 1.78508$

5) $u(1, 0.8) = 3.6499$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 42

Exercise 1

Given the system

$-2 u_1^3 + 3 x y u_3 - u_2 u_3^2 == 110$
$3 x u_2 + 2 y u_2 u_3 == -81$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-1, 5, -4, -3, 3, 1, -3)$. Compute if possible $\frac{\partial y}{\partial u_2}(-4, -3, 3, 1, -3)$.

1) $\frac{\partial y}{\partial u_2}(-4, -3, 3, 1, -3) = \frac{18}{11}$

2) $\frac{\partial y}{\partial u_2}(-4, -3, 3, 1, -3) = \frac{16}{11}$

3) $\frac{\partial y}{\partial u_2}(-4, -3, 3, 1, -3) = \frac{17}{11}$

4) $\frac{\partial y}{\partial u_2}(-4, -3, 3, 1, -3) = \frac{15}{11}$

5) $\frac{\partial y}{\partial u_2}(-4, -3, 3, 1, -3) = \frac{14}{11}$

Exercise 2

Compute the Gauss curvature for $X(u,v) =$
{Cos[u] (3 + Cos[v]) + 2 (3 + Cos[v]) Sin[u], (3 + Cos[v]) Sin[u], Sin[v]}
at the point $(u,v) = (6, 1)$.

1) $K(6, 1) = -4.36215$

2) $K(6, 1) = -3.26782$

3) $K(6, 1) = 0.0267218$

4) $K(6, 1) = 0.89194$

5) $K(6, 1) = 7.58008$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=\begin{cases} 3x & 0\le x\le 1 \\ -\frac{3x}{\pi-1}+\frac{3}{\pi-1}+3 & 1\le x\le \pi \end{cases} & 0\le x\le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=2$ and the moment $t=0.4$ by means of a Fourier series of order 11.

1) $u(2, 0.4) = -6.00132$

2) $u(2, 0.4) = 0.432$

3) $u(2, 0.4) = -4.57209$

4) $u(2, 0.4) = -6.53527$

5) $u(2, 0.4) = 5.88413$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 43

Exercise 1

Given the system

$-u^3 - 2 v^2 w - 2 w^3 - 3 u^2 x - u x^2 - v w y - x y^2 == -277$

$-3 - 3 v x - 3 u^2 y + 2 w y == 388$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (4 , -5 , -5 , -3 , 2). Compute if possible $\frac{\partial x}{\partial v}$ (-5 , -3 , 2) .

1) $\frac{\partial x}{\partial v}(-5, -3, 2) = \frac{932}{1923}$

2) $\frac{\partial x}{\partial v}(-5, -3, 2) = \frac{931}{1923}$

3) $\frac{\partial x}{\partial v}(-5, -3, 2) = \frac{311}{641}$

4) $\frac{\partial x}{\partial v}(-5, -3, 2) = \frac{935}{1923}$

5) $\frac{\partial x}{\partial v}(-5, -3, 2) = \frac{934}{1923}$

Exercise 2

Compute the Gauss curvature for X(u,v) =
$\{(1 + 2 v^2) Cos[u] - 2 (1 + 2 v^2) Sin[u], (1 + 2 v^2) Sin[u], v - (1 + 2 v^2) Cos[u] + 4 (1 + 2 v^2) Sin[u]\}$
at the point (u,v) = (1 , 4).

1) K (1 , 4) = 5.11091

2) K (1 , 4) = -4.47548×10^{-8}

3) K (1 , 4) = -2.44869

4) K (1 , 4) = -1.5306

5) K (1 , 4) = -2.77887

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{80x}{7} & 0 \le x \le \frac{7}{10} \\ 85 - 110x & \frac{7}{10} \le x \le \frac{4}{5} \\ 15x - 15 & \frac{4}{5} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{3}{10}$ and the moment $t = 0.7$ by means of a Fourier series of order 9.

1) $u\left(\frac{3}{10}, 0.7\right) = 0.00405601$

2) $u\left(\frac{3}{10}, 0.7\right) = -5.93027$

3) $u\left(\frac{3}{10}, 0.7\right) = -7.4986$

4) $u\left(\frac{3}{10}, 0.7\right) = 3.79038$

5) $u\left(\frac{3}{10}, 0.7\right) = 5.11091$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 44

Exercise 1

Given the system

$-2\,x\,y\,u_2 + 3\,y\,u_3\,u_5 == -12$
$-y - x\,u_4^2 + 2\,u_2^2\,u_5 == -1$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (3, 2, -3, 1, 0, 1, 2)$. Compute if possible $\frac{\partial x}{\partial u_3}(-3, 1, 0, 1, 2)$.

1) $\frac{\partial x}{\partial u_3}(-3, 1, 0, 1, 2) = -6$

2) $\frac{\partial x}{\partial u_3}(-3, 1, 0, 1, 2) = -4$

3) $\frac{\partial x}{\partial u_3}(-3, 1, 0, 1, 2) = -5$

4) $\frac{\partial x}{\partial u_3}(-3, 1, 0, 1, 2) = -2$

5) $\frac{\partial x}{\partial u_3}(-3, 1, 0, 1, 2) = -3$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{4\,v + v\,\mathrm{Cos}[u] + 10\,v\,\mathrm{Sin}[u],\ v + 3\,v\,\mathrm{Sin}[u],\ v + 2\,v\,\mathrm{Sin}[u]\}$ at the point $(u,v) = (3, -2)$.

1) $K(3, -2) = -5.58199$

2) $K(3, -2) = 7.7485$

3) $K(3, -2) = -4.95759$

4) $K(3, -2) = 4.36606$

5) $K(3, -2) = 0$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t)=u(2,t)=0 & 0\le t \\ u(x,0)=-2\,(x-2)\,(x-1)\,x^2 & 0\le x\le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=\frac{3}{10}$ and the moment $t=0.2$ by means of a Fourier series of order 10 .

1) $u\left(\frac{3}{10}, 0.2\right) = 5.57503$

2) $u\left(\frac{3}{10}, 0.2\right) = -5.08314$

3) $u\left(\frac{3}{10}, 0.2\right) = 0.017546$

4) $u\left(\frac{3}{10}, 0.2\right) = -4.95759$

5) $u\left(\frac{3}{10}, 0.2\right) = 7.7485$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 45

Exercise 1

Given the function

$f(x,y,z) = -9 + 6x - x^2 + 6y - y^2 - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {?, 2.30717, -0.512704}

2) We have a maximum at {1.53387, ?, 0.769057}

3) We have a maximum at {3, 3, ?}

4) We have a maximum at {?, 2.56352, 0.}

Exercise 2

Consider the vectorial field $F(x,y,z) = (yz\,(yz+z),\, xyz^2 + xz\,(yz+z),\, xy\,(y+1)\,z + xy\,(yz+z))$. Compute the potential function for this field whose potential at the origin is 4.

. Calculate the value of the potential at the point p= (-8 , -4 , -1) .

1) $\frac{322}{5}$ 2) $\frac{184}{5}$ 3) $-\frac{966}{5}$ 4) -92

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(2,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 5x & 0 \le x \le 1 \\ 10 - 5x & 1 \le x \le 2 \end{cases} & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{17}{10}$ and the moment t= 0.2 by means of a Fourier series of order 12 .

1) $u\left(\frac{17}{10}, 0.2\right) = -4.61943$

2) $u\left(\frac{17}{10}, 0.2\right) = 4.1918$

3) $u\left(\frac{17}{10}, 0.2\right) = 4.77325$

4) $u\left(\frac{17}{10}, 0.2\right) = 0.626963$

5) $u\left(\frac{17}{10}, 0.2\right) = 2.5$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 46

Exercise 1

Given the function

$f(x,y,z) = 2 - 4x + x^2 - 4y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {2, 2, ?}

2) We have a maximum at {-1.425, ?, -1.71578}

3) We have a maximum at {?, -3.35556, -1.61578}

4) We have a maximum at {-1.125, -3.55556, ?}

Exercise 2

Consider the vectorial field $F(x,y,z) = (2xy - y^2 z\cos(xy),\ x^2 - z\sin(xy) - xyz\cos(xy) + 2y,\ -y\sin(xy))$. Compute the potential function for this field whose potential at the origin is 4 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) 6.02074 2) 4.42074 3) -1.17926 4) -9.17926

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = (x-4)^2 (x-3)\, x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 3 and the moment t= 1. by means of a Fourier series of order 8 .

1) u(3 , 1.) = -0.648212

2) u(3 , 1.) = 0.00523374

3) u(3 , 1.) = 4.57538

4) u(3 , 1.) = -3.76574

5) u(3 , 1.) = -7.46636

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 47

Exercise 1

Compute the volume of the domain limited by the plane $10x + 8z == 10$ and the paraboloid $z == x^2 + y^2$.

1) 14.1941

2) 3.71148

3) 1.68976

4) 4.22803

5) 3.75852

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0,2\pi] \dashrightarrow R^2$

$$r(t) = \left\{ \frac{\left(-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos(t)\,(8\cos(t)+10)}{\sin^2(t)+1}, \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(8\cos(t)+10)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 139.538   2) 77.9381   3) 31.7381   4) 154.938

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} x & 0 \le x \le 1 \\ 2x-1 & 1 \le x \le 4 \\ 35-7x & 4 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} x & 0 \le x \le 3 \\ 3 & 3 \le x \le 4 \\ 15-3x & 4 \le x \le 5 \end{cases} & 0. \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 3$ and the moment $t = 0.2$ by means of a Fourier series of order 10 .

1) u(3 , 0.2) = −0.302894

2) u(3 , 0.2) = −0.0733791

3) u(3 , 0.2) = 5.32335

4) u(3 , 0.2) = −2.63505

5) u(3 , 0.2) = −3.84233

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 48

Exercise 1

Given the system

$3x^2 y - 3u_1 u_2 u_3 == 45$

$-3x^2 u_2 - 3y^2 u_4 == 315$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (5, -1, -2, -4, -5, -5)$. Compute if possible $\frac{\partial x}{\partial u_3}(-2, -4, -5, -5)$.

1) $\frac{\partial x}{\partial u_3}(-2, -4, -5, -5) = \frac{1}{9}$

2) $\frac{\partial x}{\partial u_3}(-2, -4, -5, -5) = \frac{7}{45}$

3) $\frac{\partial x}{\partial u_3}(-2, -4, -5, -5) = \frac{4}{45}$

4) $\frac{\partial x}{\partial u_3}(-2, -4, -5, -5) = \frac{8}{45}$

5) $\frac{\partial x}{\partial u_3}(-2, -4, -5, -5) = \frac{2}{15}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{\text{Cos}[u]\,(3 + \text{Cos}[v]) + \text{Sin}[v],\ -2\,\text{Cos}[u]\,(3 + \text{Cos}[v]) + (3 + \text{Cos}[v])\,\text{Sin}[u],\ \text{Cos}[u]\,(3 + \text{Cos}[v]) + 2\,\text{Sin}[v]\}$ at the point $(u,v) = (6, 4)$.

1) $K(6, 4) = -4.16262$

2) $K(6, 4) = -1.96214$

3) $K(6, 4) = -6.73147$

4) $K(6, 4) = -2.53128$

5) $K(6, 4) = -0.936307$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-5)\,(x-2)\,(x-1)\,x & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=3$ and the moment $t=0.7$ by means of a Fourier series of order 9 .

1) $u(3, 0.7) = 10.0269$

2) $u(3, 0.7) = -0.936307$

3) $u(3, 0.7) = -2.47745$

4) $u(3, 0.7) = -1.67536$

5) $u(3, 0.7) = 7.91435$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 49

Exercise 1

Given the function

$f(x, y, z) = -17 - x^2 + 2y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.3, ?, 3.6}

2) We have a maximum at {?, 0.7, 1.5}

3) We have a maximum at {1.2, 2.5, ?}

4) We have a maximum at {-1.5, -0.2, ?}

Exercise 2

Consider the vectorial field $F(x, y, z) = \left(-\frac{6 y^2 z^2}{x y z + 1} + y + 3, -\frac{6 x y z^2}{x y z + 1} - 6 z \log(x y z + 1) + x, -\frac{6 x y^2 z}{x y z + 1} - 6 y \log(x y z + 1)\right)$. Compute the potential function for this field whose potential at the origin is 4 .

. Calculate the value of the potential at the point p= (6 , -1 , -10) .

1) $\frac{4469}{5} - 60 \text{Log}[61]$ 2) $\frac{391}{10} - 60 \text{Log}[61]$ 3) $16 - 60 \text{Log}[61]$ 4) $-\frac{3847}{5} - 60 \text{Log}[61]$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 2 \\ -\frac{4x}{\pi-2} + \frac{8}{\pi-2} + 4 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 1. by means of a Fourier series of order 10 .

1) u (1 , 1.) = -0.902642

2) u (1 , 1.) = 2.

3) u (1 , 1.) = -2.29904

4) u (1 , 1.) = 1.47015

5) u (1 , 1.) = 4.45262

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 50

Exercise 1

Compute $\int_D (x^2 + z^3)\, dx\, dy\, dz$ for D=

$\{6 x^5 \le y^3 z^2 \le 8 x^5,\ 8 x^8 z^8 \le y^7 \le 12 x^8 z^8,\ 5 y \le x^3 \le 11 y,\ x > 0,\ y > 0,\ z > 0\}$

1) 32.7481

2) -6.82253

3) 13.6451

4) 9.55154

5) 4.09352

Exercise 2

Compute the mean curvature for X(u,v) =

$\{2 - 7 u^2 - 5 v - 2 v^2 + 4 u (1 + v),\ -4 + 14 u^2 + 11 v + 4 v^2 - u (9 + 8 v),\ -2 + 7 u^2 + 5 v + 2 v^2 - u (3 + 4 v)\}$

at the point (u,v) = (-7 , -4) .

1) H (-7 , -4) = -1.26426

2) H (-7 , -4) = -0.134738

3) H (-7 , -4) = -8.5887

4) H (-7 , -4) = 5.98388

5) H (-7 , -4) = 7.61278

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -3 (x-1)\, x^2 (x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.7 by means of a Fourier series of order 8 .

1) u (1 , 0.7) = 7.61278

2) u (1 , 0.7) = -1.26426

3) u (1 , 0.7) = -0.0143545

4) u (1 , 0.7) = 7.38804

5) u (1 , 0.7) = -3.42978

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 51

Exercise 1

Compute $\int_D (3x + z^3)\,dx\,dy\,dz$ for D=
$\{4x^2y^8z^2 \le 1 \le 12x^2y^8z^2,\ 3x^3 \le y^8z^3 \le 10x^3,\ 4 \le x z^6 \le 11,\ x > 0,\ y > 0,\ z > 0\}$

1) −1.28125

2) 0.918747

3) 1.51875

4) 0.218747

5) 0.0187465

Exercise 2

Compute the mean curvature for $X(u,v) =$
$\{2(-1 + 4u^2 - u(-2 + v) - 2v + v^2),\ v,\ -2 + 3u + 8u^2 - 2v - 2uv + 2v^2\}$
at the point $(u,v) = (8, -2)$.

1) H (8 , -2) = 5.34449

2) H (8 , -2) = −8.76887

3) H (8 , -2) = 2.91099

4) H (8 , -2) = −3.38081

5) H (8 , -2) = −0.0023605

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 3x & 0 \le x \le 1 \\ 14 - 11x & 1 \le x \le 2 \\ 8x - 24 & 2 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$
and the moment $t = 0.7$ by means of a Fourier series of order 8 .

1) u(1 , 0.7) = −8.76887

2) u(1 , 0.7) = −4.38328

3) u(1 , 0.7) = −4.43417

4) u(1 , 0.7) = −0.105753

5) u(1 , 0.7) = −2.31471

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 52

Exercise 1

Given the system

$-2v - w - uvw - 3w^3 - w^2 x + vwy == -178$

$-1 + wy - uxy - 2wy^2 == -25$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (0 , 2 , 3 , -3 , 4). Compute if possible $\frac{\partial x}{\partial w}(3, -3, 4)$.

1) $\frac{\partial x}{\partial w}(3, -3, 4) = -\frac{486}{47}$

2) $\frac{\partial x}{\partial w}(3, -3, 4) = -\frac{484}{47}$

3) $\frac{\partial x}{\partial w}(3, -3, 4) = -\frac{487}{47}$

4) $\frac{\partial x}{\partial w}(3, -3, 4) = -\frac{488}{47}$

5) $\frac{\partial x}{\partial w}(3, -3, 4) = -\frac{485}{47}$

Exercise 2

Compute the Gauss curvature for X(u,v) = {-2 v - 3 Cos[u] - 2 Sin[u], 2 v + 3 Sin[u], v + 2 Cos[u] + Sin[u]} at the point (u,v) = (0 , 9).

1) K (0 , 9) = 1.99368

2) K (0 , 9) = 0

3) K (0 , 9) = -3.68913

4) K (0 , 9) = -6.70564

5) K (0 , 9) = -5.41371

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t)=u(5,t)=0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ 35 - 14x & 2 \le x \le 3 \\ \frac{7x}{2} - \frac{35}{2} & 3 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 4$ and the moment $t = 0.3$ by means of a Fourier series of order 10 .

1) $u(4, 0.3) = -3.68913$

2) $u(4, 0.3) = 0.103208$

3) $u(4, 0.3) = -2.89212$

4) $u(4, 0.3) = 4.97242$

5) $u(4, 0.3) = -5.15922$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 53

Exercise 1

Compute the volume of the domain limited by the plane $3x + 8z == 10$ and the paraboloid $z == 6x^2 + 6y^2$.

1) 0.361449

2) 0.630275

3) 0.322305

4) 1.84253

5) 0.412905

Exercise 2

Consider the vector field $F(x,y,z) = \{-x y + \mathrm{Sin}[2 y^2 - z^2],\ 8 x z + \mathrm{Sin}[2 z^2],\ e^{-x^2+2y^2} + 2 x y z\}$ and the surface

$$S \equiv \left(\frac{-6+x}{6}\right)^2 + \left(\frac{y}{2}\right)^2 + \left(\frac{2+z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 0.6   2) 1.   3) -0.7   4) 0.

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 1 \\ \frac{7x}{2} - \frac{3}{2} & 1 \le x \le 3 \\ -\frac{9x}{\pi-3} + \frac{27}{\pi-3} + 9 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t} u(x,0) = \begin{cases} -6x & 0 \le x \le 1 \\ \frac{6x}{\pi-1} - \frac{6}{\pi-1} - 6 & 1 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \mathrm{True} \end{cases}$$

Compute the position of the string at x= 2 and the moment t= 0.7 by means of a Fourier series of order 8.

1) u(2 , 0.7) = 7.96872

2) u(2 , 0.7) = 7.08849

3) u(2 , 0.7) = 0.206489

4) u(2 , 0.7) = -7.46227

5) u(2 , 0.7) = -4.45583

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 54

Exercise 1

Compute $\int_D (2\,x\,y)\,dx\,dy\,dz$ for D=

$\{8\,x^2\,y^8 \le z^6 \le 12\,x^2\,y^8,\ 4\,y^3\,z^5 \le x^8 \le 13\,y^3\,z^5,\ 9 \le x^8\,y^4\,z \le 18,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.501927

2) 0.701927

3) 0.00192659

4) 1.40193

5) 0.101927

Exercise 2

Consider the vectorial field $F(x, y, z) = ($
$x\,y^2\,z\cos(x\,y\,z) + y\sin(x\,y\,z) - 2,\ x^2\,y\,z\cos(x\,y\,z) + x\sin(x\,y\,z) + 2,\ x^2\,y^2\cos(x\,y\,z)$
$)$. Compute the potential function for this field whose potential at the origin is 0 .

. Calculate the value of the potential at the point p = (0 , 4 , 7) .

1) 8 2) $-\frac{72}{5}$ 3) $-\frac{68}{5}$ 4) $\frac{176}{5}$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2\,x & 0 \le x \le 1 \\ -\frac{2\,x}{\pi-1} + \frac{2}{\pi-1} + 2 & 1 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x = 2
and the moment t = 0.2 by means of a Fourier series of order 9 .

1) u(2 , 0.2) = 0.447407

2) u(2 , 0.2) = -1.64859

3) u(2 , 0.2) = 4.1855

4) u(2 , 0.2) = 4.60211

5) u(2 , 0.2) = 0.994396

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 55

Exercise 1

Compute $\int_D (y^5)\, dx\, dy\, dz$ for D=

$\{6 x^7 z^7 \le y^2 \le 10 x^7 z^7,\ 2 x^3 y^3 z^8 \le 1 \le 11 x^3 y^3 z^8,\ 9 \le x^4 y^5 z^5 \le 15,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.146595

2) 1.8466

3) 1.9466

4) -1.0534

5) 1.4466

Exercise 2

Consider the vectorial field $F(x, y, z) = (x^2 y^3 z^2 + 2 x y^2 z^2 (x y - y z) - y, x^2 y^2 z^2 (x - z) + 2 x^2 y z^2 (x y - y z) - x, 2 x^2 y^2 z (x y - y z) - x^2 y^3 z^2)$. Compute the potential function for this field whose potential at the origin is -3 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) 0.75 2) -3.25 3) 1.95 4) -8.85

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2 (x-2)(x-1) x (x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
and the moment t= 0.2 by means of a Fourier series of order 12 .

1) u (1 , 0.2) = -2.61123

2) u (1 , 0.2) = -6.12651

3) u (1 , 0.2) = -5.86333

4) u (1 , 0.2) = -8.69018

5) u (1 , 0.2) = -0.0169162

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 56

Exercise 1

Given the system

$-3 v^2 + 2 u^2 w + 3 v x - 3 u x^2 - u w y + 3 w x y == -65$

$3 + 2 v w - 3 w x^2 + 2 x y == -15$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point $p=(x, y, u, v, w) = (2, 1, -2, -5, 1)$. Compute if possible $\frac{\partial x}{\partial u}(-2, -5, 1)$.

1) $\frac{\partial x}{\partial u}(-2, -5, 1) = \frac{11}{16}$

2) $\frac{\partial x}{\partial u}(-2, -5, 1) = \frac{21}{32}$

3) $\frac{\partial x}{\partial u}(-2, -5, 1) = \frac{3}{4}$

4) $\frac{\partial x}{\partial u}(-2, -5, 1) = \frac{25}{32}$

5) $\frac{\partial x}{\partial u}(-2, -5, 1) = \frac{23}{32}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{8 u^2 - u (1 + 9 v) - 2 (2 + 4 v + v^2), 4 - 8 u^2 + 9 v + 9 u v + 2 v^2, 8 u^2 - u (2 + 9 v) - 2 (2 + 4 v + v^2)\}$ at the point $(u,v) = (-2, 4)$.

1) $K(-2, 4) = -2.42396$

2) $K(-2, 4) = -3.46652$

3) $K(-2, 4) = -9.45314 \times 10^{-7}$

4) $K(-2, 4) = 6.36576$

5) $K(-2, 4) = -4.3733$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{x}{3} & 0 \le x \le 3 \\ x-4 & 3 \le x \le 4 \end{cases} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 3$ and the moment $t = 0.7$ by means of a Fourier series of order 11 .

1) $u(3, 0.7) = 4.69928$

2) $u(3, 0.7) = -0.0963413$

3) $u(3, 0.7) = -2.42396$

4) $u(3, 0.7) = -7.73536$

5) $u(3, 0.7) = 2.20093$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 57

Exercise 1

Given the function

$f(x, y, z) = 7 + x^2 - 2y + y^2 - 2z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, 1.3, 1.2}

2) We have a minimum at {-0.3, ?, 0.7}

3) We have a minimum at {0.5, ?, 1.5}

4) We have a minimum at {0.3, ?, 1.4}

Exercise 2

Compute the Gauss curvature for X(u,v) = {Cos[u] (3 + Cos[v]), 6 Cos[u] (3 + Cos[v]) + (3 + Cos[v]) Sin[u] - 3 Sin[v], -2 Cos[u] (3 + Cos[v]) + Sin[v]} at the point (u,v) = (1, 5).

1) K(1, 5) = 2.64407

2) K(1, 5) = 0.00828164

3) K(1, 5) = 6.33028

4) K(1, 5) = 8.38243

5) K(1, 5) = 8.35111

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = 3(x-2)\,x^2\,(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.6 by means of a Fourier series of order 12.

1) u(2, 0.6) = 3.01411

2) u(2, 0.6) = 1.46893

3) u(2, 0.6) = 4.65691

4) u(2, 0.6) = -2.37959

5) u(2, 0.6) = -4.1696

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 58

Exercise 1

Compute the volume of the domain limited by the plane $10x + 3z == 3$ and the paraboloid $z == 2x^2 + 2y^2$.

1) 6.24588

2) 1.07375

3) 13.3644

4) 4.4821

5) 2.49448

Exercise 2

Consider the vector field $F(x,y,z) = \left\{-5z - 8xz - \mathrm{Sin}\left[y^2 - z^2\right], 9z - 5yz + \mathrm{Sin}\left[x^2 - z^2\right], e^{-2x^2+2y^2} - xz - 5yz\right\}$ and the surface

$$S \equiv \left(\frac{-5+x}{3}\right)^2 + \left(\frac{-5+y}{3}\right)^2 + \left(\frac{-4+z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −617.954 2) 10 511.4 3) −21 640.2 4) −6182.65

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -x & 0 \le x \le 1 \\ 4x - 5 & 1 \le x \le 2 \\ 5 - x & 2 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ \frac{\partial}{\partial t}u(x,0) = -2(x-5)(x-3)(x-1)x^2 & 0. \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1 and the moment t= 0.3 by means of a Fourier series of order 10 .

1) u (1 , 0.3) = 2.399

2) u (1 , 0.3) = −0.416888

3) u (1 , 0.3) = 1.99658

4) u (1 , 0.3) = 5.28316

5) u (1 , 0.3) = −6.18431

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 59

Exercise 1

Given the system

$u_1 - 3 x u_1^2 + u_2 - 3 y u_3 + 2 x u_3 u_4 + 3 u_3 u_4^2 == 217$

$-2 x^2 u_2 + 3 y^2 u_2 - 3 u_3^2 - 2 x u_2 u_4 == 9$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (-3, 1, 5, -1, -2, -1)$. Compute if possible $\frac{\partial x}{\partial u_1}(5, -1, -2, -1)$.

1) $\frac{\partial x}{\partial u_1}(5, -1, -2, -1) = \frac{92}{85}$

2) $\frac{\partial x}{\partial u_1}(5, -1, -2, -1) = \frac{93}{85}$

3) $\frac{\partial x}{\partial u_1}(5, -1, -2, -1) = \frac{91}{85}$

4) $\frac{\partial x}{\partial u_1}(5, -1, -2, -1) = \frac{19}{17}$

5) $\frac{\partial x}{\partial u_1}(5, -1, -2, -1) = \frac{94}{85}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{v \,\mathrm{Cos}[u],\ v + v\,\mathrm{Cos}[u] + v\,\mathrm{Sin}[u],\ 2 v + 2 v\,\mathrm{Cos}[u] + v\,\mathrm{Sin}[u]\}$ at the point $(u,v) = (0, 7)$.

1) $K(0, 7) = -4.95538$

2) $K(0, 7) = 2.51894$

3) $K(0, 7) = -3.00399$

4) $K(0, 7) = -2.286$

5) $K(0, 7) = 0$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = (x-4)(x-2)x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.8 by means of a Fourier series of order 8 .

1) u(2 , 0.8) = -1.26789

2) u(2 , 0.8) = -8.43559

3) u(2 , 0.8) = 0

4) u(2 , 0.8) = 2.51894

5) u(2 , 0.8) = 1.84686

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 60

Exercise 1

Compute $\int_D (x + x^2)\, dx\, dy\, dz$ for D=

$\{7 y^3 z^2 \le x^4 \le 13 y^3 z^2,\ 2 x^9 \le y^9 z^2 \le 10 x^9,\ 5 x^3 z \le y^9 \le 11 x^3 z,\ x > 0,\ y > 0,\ z > 0\}$

1) 2.00004

2) 1.90004

3) 0.700036

4) 1.10004

5) 0.0000361254

Exercise 2

Compute the mean curvature for X(u,v) =

{Cos[u] (3 + Cos[v]) + 2 (3 + Cos[v]) Sin[u] - Sin[v], (3 + Cos[v]) Sin[u], Sin[v]}

at the point (u,v) = (3 , 5).

1) H (3 , 5) = -7.23036

2) H (3 , 5) = -6.29396

3) H (3 , 5) = 1.00082

4) H (3 , 5) = 8.3389

5) H (3 , 5) = 5.77446

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 2\,(x-2)\,(x-1)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = 2\,(x-3)\,(x-2)\,x^2\,(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x = 1

and the moment t = 0.4 by means of a Fourier series of order 9 .

1) u (1 , 0.4) = 4.24317

2) u (1 , 0.4) = 4.40609

3) u (1 , 0.4) = -5.68088

4) u (1 , 0.4) = 5.91531

5) u (1 , 0.4) = -1.8239

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 61

Exercise 1

Compute $\int_D (y + z)\,dx\,dy\,dz$ for D=
$\{3 x^8 y^3 \le z^2 \le 4 x^8 y^3,\ 8 x^7 \le z^7 \le 11 x^7,\ 9 x^3 \le y^4 z \le 16 x^3,\ x > 0,\ y > 0,\ z > 0\}$

1) -0.899291

2) 1.90071

3) -0.599291

4) 0.000708989

5) 1.30071

Exercise 2

Consider the vector field $F(x,y,z) = \{x z + \mathrm{Sin}[z^2],\ 5 x y z + \mathrm{Cos}[2 x^2 - 2 z^2],\ 6 + 9 z - \mathrm{Sin}[x^2 + 2 y^2]\}$ and the surface

$$S \equiv \left(\frac{7 + x}{9}\right)^2 + \left(\frac{-4 + y}{4}\right)^2 + \left(\frac{-9 + z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) -241 847. 2) 241 847. 3) -403 079. 4) -765 850.

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 25 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -4x & 0 \le x \le 1 \\ x - 5 & 1 \le x \le 2 \\ \frac{3x}{\pi-2} - \frac{6}{\pi-2} - 3 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.3 by means of a Fourier series of order 8 .

1) u(2 , 0.3) = 0.994879

2) u(2 , 0.3) = -2.29568

3) u(2 , 0.3) = -0.123982

4) u(2 , 0.3) = 3.20175

5) u(2 , 0.3) = -3.95028

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 62

Exercise 1

Given the system

$-v^3 - 2 w^2 x + u^2 y == -35$

$3 v w^2 + x - 2 u x + 3 w x^2 + v w y + v x y == -15$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (1 , -2 , 3 , -1 , 3). Compute if possible $\frac{\partial x}{\partial u}(3, -1, 3)$.

1) $\frac{\partial x}{\partial u}(3, -1, 3) = \frac{8}{7}$

2) $\frac{\partial x}{\partial u}(3, -1, 3) = \frac{22}{21}$

3) $\frac{\partial x}{\partial u}(3, -1, 3) = \frac{25}{21}$

4) $\frac{\partial x}{\partial u}(3, -1, 3) = \frac{26}{21}$

5) $\frac{\partial x}{\partial u}(3, -1, 3) = \frac{23}{21}$

Exercise 2

Compute the Gauss curvature for X(u,v) = {v - v Sin[u], v Sin[u], 2 v - v Cos[u] - v Sin[u]} at the point (u,v) = (1 , -5).

1) K (1 , -5) = 8.26069

2) K (1 , -5) = 0

3) K (1 , -5) = 6.32033

4) K (1 , -5) = -6.38339

5) K (1 , -5) = 8.51142

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{7x}{2} & 0 \le x \le 2 \\ \frac{7x}{\pi-2} - \frac{14}{\pi-2} - 7 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.3 by means of a Fourier series of order 12 .

1) u (1 , 0.3) = 0.3954

2) u (1 , 0.3) = −3.15388

3) u (1 , 0.3) = 8.44373

4) u (1 , 0.3) = −6.38339

5) u (1 , 0.3) = −5.56993

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 63

Exercise 1

Given the function

$f(x, y, z) = -14 + 6x - x^2 - y^2 - z^2$ defined over the domain $D \equiv \frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {-3., 0., ?}

2) We have a minimum at {?, 0.4, 0.3}

3) We have a minimum at {-2.6, ?, -0.4}

4) We have a minimum at {?, -0.5, -0.5}

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{Cos[u], -2v + 2Cos[u] + Sin[u], v - Cos[u]\}$ at the point $(u,v) = (1, -7)$.

1) K (1 , -7) = -8.40913

2) K (1 , -7) = -3.31287

3) K (1 , -7) = -1.21116

4) K (1 , -7) = -8.68075

5) K (1 , -7) = 0

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(3,t) = 0 & 0 \le t \\ u(x,0) = -((x-3)(x-2)(x-1)x) & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.6$ by means of a Fourier series of order 10 .

1) u(2 , 0.6) = 0.3

2) u(2 , 0.6) = -1.60698

3) u(2 , 0.6) = 2.06719

4) u(2 , 0.6) = -1.73942

5) u(2 , 0.6) = -4.67174

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 64

Exercise 1

Compute the volume of the domain limited by the plane $10x + 3z == 4$ and the paraboloid $z == 4x^2 + 4y^2$.

1) 1.61473

2) 6.15471

3) 0.381505

4) 0.632591

5) 6.42362

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] ----\rightarrow R^2$

$$r(t) = \left\{ \frac{\left(-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+9)}{\sin^2(t)+1}, \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+9)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 194.26 2) 102.46 3) 20.8602 4) 163.66

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 1 \\ \frac{5}{2} - \frac{x}{2} & 1 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ \frac{\partial}{\partial t}u(x,0) = -3(x-5)(x-2)(x-1)x^2 & 0. \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$ and the moment $t = 0.7$ by means of a Fourier series of order 8 .

1) u(1 , 0.7) = 37.9657

2) u(1 , 0.7) = -5.68978

3) u(1 , 0.7) = -2.22466

4) u(1 , 0.7) = -0.911611

5) u(1 , 0.7) = 7.05398

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 65

Exercise 1

Given the function

$f(x, y, z) = 3 - 2x + x^2 - 4y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-1.64164, ?, 0.3}

2) We have a maximum at {-1.34164, ?, 0.}

3) We have a maximum at {1, ?, 0}

4) We have a maximum at {-1.84164, -3.18328, ?}

Exercise 2

Compute the Gauss curvature for X(u,v) = $\{(1+v^2)\,\mathrm{Cos}[u],\ (1+v^2)\,\mathrm{Sin}[u],\ v+(1+v^2)\,\mathrm{Cos}[u]\}$ at the point (u,v) = (6 , 10).

1) K (6 , 10) = -2.81037×10^{-8}

2) K (6 , 10) = -8.88757

3) K (6 , 10) = 2.75234

4) K (6 , 10) = 4.56541

5) K (6 , 10) = 3.94236

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 2 \\ -\frac{4x}{\pi-2} + \frac{8}{\pi-2} + 4 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.9 by means of a Fourier series of order 10 .

1) u(2 , 0.9) = 2.00006

2) u(2 , 0.9) = 2.53634

3) u(2 , 0.9) = -2.6036

4) u(2 , 0.9) = -2.46924

5) u(2 , 0.9) = 1.23346

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 66

Exercise 1

Compute $\int_D (x^2 + y^3)\,dx\,dy\,dz$ for D=

$\{4x^3 z^9 \le y^4 \le 7x^3 z^9,\ 6x^3 z^5 \le y^8 \le 10x^3 z^5,\ 4x^6 z^2 \le y^4 \le 7x^6 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.00150755

2) −1.29849

3) 0.201508

4) 1.80151

5) −1.89849

Exercise 2

Consider the vector field F(x,y,z) =

$\{e^{2y^2-2z^2} + 8xy,\ -3yz + \mathrm{Cos}[x^2],\ 4x + \mathrm{Sin}[y^2]\}$ and the surface

$S \equiv \left(\frac{-1+x}{9}\right)^2 + \left(\frac{-3+y}{7}\right)^2 + \left(\frac{8+z}{7}\right)^2 = 1$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −239 403. 2) −177 336. 3) 88 668.3 4) −186 202.

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ 10 - \frac{3x}{2} & 2 \le x \le 4 \\ 20 - 4x & 4 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
and the moment t= 0.3 by means of a Fourier series of order 11 .

1) u (1 , 0.3) = 3.88631

2) u (1 , 0.3) = −0.815142

3) u (1 , 0.3) = 0.252446

4) u (1 , 0.3) = −2.97678

5) u (1 , 0.3) = −0.182033

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 67

Exercise 1

Given the function

$f(x,y,z) = 1 - x^2 + 4y - y^2 + 4z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0, 2, ?}

2) We have a maximum at {?, 2.4, 1.4}

3) We have a maximum at {-0.6, ?, 1.2}

4) We have a maximum at {?, 3., 1.}

Exercise 2

Consider the vectorial field $F(x,y,z) = \left(-2xy^2 + yz(-2yz - z)\,e^{xyz} + 2xy,\ -2x^2y + x^2 + xz(-2yz - z)\,e^{xyz} - 2z\,e^{xyz},\ (-2y - 1)\,e^{xyz} + xy(-2yz - z)\,e^{xyz}\right)$. Compute the potential function for this field whose potential at the origin is 3 .

. Calculate the value of the potential at the point $p = (-4, 4, 1)$.

1) $-189 - \frac{9}{e^{16}}$ 2) $571 - \frac{9}{e^{16}}$ 3) $-854 - \frac{9}{e^{16}}$ 4) $191 - \frac{9}{e^{16}}$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 2x & 0 \le x \le 3 \\ -\frac{6x}{\pi-3} + \frac{18}{\pi-3} + 6 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.4 by means of a Fourier series of order 10 .

1) u(2, 0.4) = 3.00157

2) u(2, 0.4) = -0.0393287

3) u(2, 0.4) = -4.2663

4) u(2, 0.4) = -3.38086

5) u(2, 0.4) = -0.988095

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 68

Exercise 1

Compute $\int_D (3x + y)\,dx\,dy\,dz$ for D=
$\{9 y^6 \le x^3 z^2 \le 18 y^6,\ 1 \le x^7 y z^3 \le 2,\ 8 y^8 z^9 \le x^2 \le 10 y^8 z^9,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.7007

2) 1.9007

3) 1.5007

4) 0.000699224

5) 0.800699

Exercise 2

Compute the mean curvature for $X(u,v) =$
$\{u,\ -7 - 5u^2 - 2u(-2 + v) - 5v + 7v^2,\ -7 - 5u^2 - 2u(-2 + v) - 6v + 7v^2\}$
at the point $(u,v) = (3, -9)$.

1) H (3 , -9) = -1.90983

2) H (3 , -9) = 0.0249283

3) H (3 , -9) = -1.47127

4) H (3 , -9) = -7.46242

5) H (3 , -9) = 6.32488

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = 3(x-3)(x-2)x^2(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} 3x & 0 \le x \le 2 \\ 6 & 2 \le x \le 3 \\ -\frac{6x}{\pi-3} + \frac{18}{\pi-3} + 6 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 0.9 by means of a Fourier series of order 11 .

1) u(1 , 0.9) = 3.1808

2) u(1 , 0.9) = 1.93312

3) u(1 , 0.9) = 0.646913

4) u(1 , 0.9) = -0.263191

5) u(1 , 0.9) = -5.08921

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 69

Exercise 1

Compute $\int_D (3x + x^3)\,dx\,dy\,dz$ for D=

$\{5 \le x^4 y^7 z^4 \le 12,\ 8 y^8 \le x^7 z^3 \le 10 y^8,\ 2 x^8 y^5 z^2 \le 1 \le 7 x^8 y^5 z^2,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.00330902

2) −1.89669

3) 1.40331

4) 1.70331

5) −0.196691

Exercise 2

Consider the vectorial field $F(x, y, z) = ($
$x^3 y^4 + 3 x^2 y^3 (x y + 2 y z),\ x^3 y^3 (x + 2 z) + 3 x^3 y^2 (x y + 2 y z),\ 2 x^3 y^4$
). Compute the potential function for this field whose potential at the origin is 1 .

. Calculate the value of the potential at the point $p = (5, 3, -9)$.

1) −131 624   2) −526 496   3) −460 684   4) −658 120

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = -((x-5)(x-4)x) & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.9 by means of a Fourier series of order 11 .

1) u(2 , 0.9) = 8.45657

2) u(2 , 0.9) = −5.23945

3) u(2 , 0.9) = −5.8807

4) u(2 , 0.9) = −0.031258

5) u(2 , 0.9) = −8.36429

Further Mathematics - Degree in Engineering - 2024/2025 Exam January-Call - computers for serial number: 70

Exercise 1

Given the function

$f(x, y, z) = 10 - 2x + x^2 - 6y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {?, 2.7, 0.9}

2) We have a minimum at {?, 3.9, -0.3}

3) We have a minimum at {?, 3, 0}

4) We have a minimum at {?, 3.6, 0.9}

Exercise 2

Consider the vectorial field $F(x, y, z) = \left(\frac{x y^2 z}{x y z + 1} + y \log(x y z + 1), \frac{x^2 y z}{x y z + 1} + x \log(x y z + 1) - 3, \frac{x^2 y^2}{x y z + 1}\right)$. Compute the potential function for this field whose potential at the origin is 3.

. Calculate the value of the potential at the point $p = (-5, -9, 5)$.

1) $30 + 45 \text{Log}[226]$ 2) $-\frac{10347}{10} + 45 \text{Log}[226]$

3) $\frac{5487}{10} + 45 \text{Log}[226]$ 4) $\frac{2061}{5} + 45 \text{Log}[226]$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(4,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -x & 0 \le x \le 1 \\ \frac{x}{3} - \frac{4}{3} & 1 \le x \le 4 \end{cases} & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.9$ by means of a Fourier series of order 9.

1) u(1, 0.9) = -0.500022

2) u(1, 0.9) = 0.266648

3) u(1, 0.9) = -1.5925

4) u(1, 0.9) = -1.64072

5) u(1, 0.9) = 4.44106

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 71

Exercise 1

Given the system

$3u^3 - 2u^2 v - 2uv^2 + 2x^3 - 2uvy == -342$

$2uv^2 - 2uy + 2vxy + vy^2 == -15$

determine if it is possible to solve for variables x, y in terms of variables u, v arround the point $p = (x, y, u, v) = (-3, -3, -4, 3)$. Compute if possible $\frac{\partial x}{\partial v}(-4, 3)$.

1) $\frac{\partial x}{\partial v}(-4, 3) = \frac{31}{45}$

2) $\frac{\partial x}{\partial v}(-4, 3) = \frac{19}{27}$

3) $\frac{\partial x}{\partial v}(-4, 3) = \frac{91}{135}$

4) $\frac{\partial x}{\partial v}(-4, 3) = \frac{94}{135}$

5) $\frac{\partial x}{\partial v}(-4, 3) = \frac{92}{135}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{v\,\mathrm{Cos}[u] - 2v\,\mathrm{Sin}[u],\ v\,\mathrm{Sin}[u],\ v - 2v\,\mathrm{Cos}[u] + 4v\,\mathrm{Sin}[u]\}$ at the point $(u,v) = (6, 8)$.

1) $K(6, 8) = -8.52921$

2) $K(6, 8) = 6.04176$

3) $K(6, 8) = 5.11862$

4) $K(6, 8) = 2.1208$

5) $K(6, 8) = 0$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = -((x-3)(x-2)(x-1)x) & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.9$ by means of a Fourier series of order 8.

1) $u(1, 0.9) = 2.81498$

2) $u(1, 0.9) = 0.0429565$

3) $u(1, 0.9) = -5.89744$

4) $u(1, 0.9) = 3.11296$

5) $u(1, 0.9) = 2.1208$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 72

Exercise 1

Compute the volume of the domain limited by the plane $10\,x + 4\,z == 6$ and the paraboloid $z == 5\,x^2 + 5\,y^2$.

1) 1.03206

2) 0.881709

3) 3.6863

4) 4.93609

5) 2.82524

Exercise 2

Consider the vector field $F(x,y,z) = \{-7\,x\,z + \mathrm{Cos}[2\,y^2],\ 5\,y\,z - \mathrm{Sin}[2\,z^2],\ -9 + 4\,x + \mathrm{Cos}[y^2]\}$ and the surface

$$S \equiv \left(\frac{-2 + x}{3}\right)^2 + \left(\frac{-4 + y}{7}\right)^2 + \left(\frac{-4 + z}{9}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 2534.15 2) -5066.65 3) -6333.45 4) 7601.35

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 5x & 0 \le x \le 1 \\ 10-5x & 1 \le x \le 2 \end{cases} & 0 \le x \le 2 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} 3x & 0 \le x \le 1 \\ 6-3x & 1 \le x \le 2 \end{cases} & 0. \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{6}{5}$ and the moment $t = 0.9$ by means of a Fourier series of order 11 .

1) $u\left(\frac{6}{5}, 0.9\right) = 0.777315$

2) $u\left(\frac{6}{5}, 0.9\right) = -7.78745$

3) $u\left(\frac{6}{5}, 0.9\right) = -7.88413$

4) $u\left(\frac{6}{5}, 0.9\right) = -7.31095$

5) $u\left(\frac{6}{5}, 0.9\right) = 2.76218$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 73

Exercise 1

Given the system

$2u - 3u^2 + 3u^3 - x^2 + 3x^3 - 3y + 3uy - u^2y + 2y^2 - xy^2 + y^3 == -424$

$-3 + x + 2x^2 + 3x^3 + 2y - 3uy + 3xy - 3uy^2 - 2y^3 == 453$

determine if it is possible to solve for variables x, y in terms of variable u arround the point $p = (x, y, u) = (4, 4, -5)$. Compute if possible $\frac{\partial x}{\partial u}(-5)$.

1) $\frac{\partial x}{\partial u}(-5) = -\frac{15713}{8263}$

2) $\frac{\partial x}{\partial u}(-5) = -\frac{15714}{8263}$

3) $\frac{\partial x}{\partial u}(-5) = -\frac{15715}{8263}$

4) $\frac{\partial x}{\partial u}(-5) = -\frac{15717}{8263}$

5) $\frac{\partial x}{\partial u}(-5) = -\frac{15716}{8263}$

Exercise 2

Compute the Gauss curvature for X(u,v) =
{2 Cos[u] (3 + Cos[v]) - (3 + Cos[v]) Sin[u], -Cos[u] (3 + Cos[v]) + (3 + Cos[v]) Sin[u], Sin[v]}
at the point (u,v) = (6, 6).

1) K(6, 6) = -5.46441

2) K(6, 6) = -4.30028

3) K(6, 6) = 0.567182

4) K(6, 6) = -2.00082

5) K(6, 6) = 5.62264

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<2,\ 0<t \\ u(0,t) = u(2,t) = 0 & 0 \le t \\ u(x,0) = 2\,(x-2)^2\,(x-1)\,x & 0 \le x \le 2 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{3}{2}$ and the moment $t = 0.9$ by means of a Fourier series of order 12 .

1) $u\left(\frac{3}{2}, 0.9\right) = 5.98502$

2) $u\left(\frac{3}{2}, 0.9\right) = 4.1082$

3) $u\left(\frac{3}{2}, 0.9\right) = -0.0215293$

4) $u\left(\frac{3}{2}, 0.9\right) = 1.30669$

5) $u\left(\frac{3}{2}, 0.9\right) = -5.46441$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 74

Exercise 1

Compute the volume of the domain limited by the plane $2x + 6z == 7$ and the paraboloid $z == 2x^2 + 2y^2$.

1) 0.939919

2) 0.530404

3) 0.8303

4) 1.09462

5) 3.79128

Exercise 2

Consider the vector field F(x,y,z) = $\{6xy - 4yz - \mathrm{Sin}[z^2], 3y - 8xz + \mathrm{Cos}[z^2], e^{2x^2+y^2} + 4y - 3z\}$ and the surface

$S \equiv \left(\frac{-2+x}{4}\right)^2 + \left(\frac{9+y}{6}\right)^2 + \left(\frac{-7+z}{3}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −9771.22 2) −16 286. 3) 3258.38 4) −57 003.5

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{2x}{3} & 0 \le x \le 3 \\ \frac{2x}{\pi-3} - \frac{6}{\pi-3} - 2 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -2x & 0 \le x \le 2 \lor 2 \le x \le 3 \\ \frac{6x}{\pi-3} - \frac{18}{\pi-3} - 6 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1 and the moment t= 0.8 by means of a Fourier series of order 9.

1) u(1, 0.8) = 6.33049

2) u(1, 0.8) = −4.01609

3) u(1, 0.8) = 7.34124

4) u(1, 0.8) = 1.58241

5) u(1, 0.8) = 0.308309

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 75

Exercise 1

Given the system

$-3\,x\,y^2 == 240$

$-3\,u^2\,w + 2\,u\,v\,x - 2\,w\,y^2 == 360$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p = (x , y , u , v , w) = (-5 , -4 , -4 , 3 , -3) . Compute if possible $\frac{\partial x}{\partial u}(-4, 3, -3)$.

1) $\frac{\partial x}{\partial u}(-4, 3, -3) = -\frac{83}{4}$

2) $\frac{\partial x}{\partial u}(-4, 3, -3) = -\frac{81}{4}$

3) $\frac{\partial x}{\partial u}(-4, 3, -3) = -21$

4) $\frac{\partial x}{\partial u}(-4, 3, -3) = -\frac{85}{4}$

5) $\frac{\partial x}{\partial u}(-4, 3, -3) = -\frac{41}{2}$

Exercise 2

Compute the Gauss curvature for X(u,v) = {v + v Cos[u], -v - v Cos[u] + v Sin[u], v} at the point (u,v) = (4 , -5) .

1) K (4 , -5) = -8.77251

2) K (4 , -5) = 5.12992

3) K (4 , -5) = 0

4) K (4 , -5) = 4.69637

5) K (4 , -5) = -1.68733

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{5x}{3} & 0 \le x \le 3 \\ -\frac{5x}{\pi-3} + \frac{15}{\pi-3} + 5 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 1$ and the moment $t = 0.9$ by means of a Fourier series of order 9.

1) $u(1, 0.9) = -1.68733$

2) $u(1, 0.9) = 1.09577$

3) $u(1, 0.9) = -3.16967$

4) $u(1, 0.9) = 4.01199$

5) $u(1, 0.9) = -5.30497$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 76

Exercise 1

Given the system

$2\,x\,y\,u_2 + 2\,u_1\,u_4^2 == -88$

$3\,x\,y^2 == -54$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-2, -3, -5, -4, -3, -2, -1)$. Compute if possible $\frac{\partial x}{\partial u_5}(-5, -4, -3, -2, -1)$.

1) $\frac{\partial x}{\partial u_5}(-5, -4, -3, -2, -1) = 0$

2) $\frac{\partial x}{\partial u_5}(-5, -4, -3, -2, -1) = 1$

3) $\frac{\partial x}{\partial u_5}(-5, -4, -3, -2, -1) = 3$

4) $\frac{\partial x}{\partial u_5}(-5, -4, -3, -2, -1) = 4$

5) $\frac{\partial x}{\partial u_5}(-5, -4, -3, -2, -1) = 2$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{2\,v + v\,\mathrm{Cos}[u] + v\,\mathrm{Sin}[u],\ v\,\mathrm{Sin}[u],\ v\}$ at the point $(u,v) = (0, -4)$.

1) $K(0, -4) = 6.80638$

2) $K(0, -4) = -0.517856$

3) $K(0, -4) = 0$

4) $K(0, -4) = 7.94007$

5) $K(0, -4) = 2.24287$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{x}{3} & 0 \le x \le 3 \\ -\frac{x}{\pi-3} + \frac{3}{\pi-3} + 1 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 0.2$ by means of a Fourier series of order 8 .

1) $u(2, 0.2) = -1.49834$

2) $u(2, 0.2) = 0.281574$

3) $u(2, 0.2) = -4.35531$

4) $u(2, 0.2) = 6.80638$

5) $u(2, 0.2) = -0.517856$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 77

Exercise 1

Compute $\int_D (x + y^2)\, dx\, dy\, dz$ for D=
$\{9\, x\, y^5\, z^5 \le 1 \le 12\, x\, y^5\, z^5,\ 8\, y^3\, z \le x^9 \le 13\, y^3\, z,\ 7\, y \le x^8\, z^7 \le 14\, y,\ x > 0,\ y > 0,\ z > 0\}$

1) 1.90028

2) 1.10028

3) 1.90028

4) 1.40028

5) 0.000282507

Exercise 2

Compute the mean curvature for X(u,v) =
$\{u + 8\, u^2 + 14\, u\, v + 2\, (-9 - 6\, v + 7\, v^2),\ 9 - 4\, u^2 + 6\, v - 7\, u\, v - 7\, v^2,\ -9 + 4\, u^2 - 7\, v + 7\, u\, v + 7\, v^2\}$
at the point (u,v) = (-9 , -8) .

1) H (-9 , -8) = -6.14992

2) H (-9 , -8) = 6.81761

3) H (-9 , -8) = -1.07807

4) H (-9 , -8) = 0.0392344

5) H (-9 , -8) = -4.71657

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<4,\ 0<t \\ u(0,t) = u(4,t) = 0 & 0 \le t \\ u(x,0) = (x-4)^2\,(x-3)\,x & 0 \le x \le 4 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 3
and the moment t= 1. by means of a Fourier series of order 9 .

1) u (3 , 1.) = -5.275

2) u (3 , 1.) = 1.37161

3) u (3 , 1.) = -6.14992

4) u (3 , 1.) = -6.72372

5) u (3 , 1.) = -0.62987

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 78

Exercise 1

Compute the volume of the domain limited by the plane $10x + 4z == 8$ and the paraboloid $z == 10x^2 + 10y^2$.

1) 0.848724

2) 0.408734

3) 2.25189

4) 0.157563

5) 0.730328

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] \dashrightarrow R^2$

$$r(t) = \left\{ \frac{\left(-\frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+8)}{\sin^2(t)+1}, \frac{\left(\frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(5\cos(t)+8)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent the curve to check whether it has intersection points.

1) 136.46 2) 51.4602 3) 85.4602 4) 119.46

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -x & 0 \le x \le 2 \\ 4x - 10 & 2 \le x \le 3 \\ 5 - x & 3 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ \frac{\partial}{\partial t}u(x,0) = -(x-5)^2 (x-2)(x-1)x & 0. \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$ and the moment $t = 0.6$ by means of a Fourier series of order 11 .

1) u (1 , 0.6) = -8.28847

2) u (1 , 0.6) = 4.91495

3) u (1 , 0.6) = 2.28795

4) u (1 , 0.6) = -7.43281

5) u (1 , 0.6) = -2.5737

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 79

Exercise 1

Compute the volume of the domain limited by the plane $6x + 5z == 9$ and the paraboloid $z == 2x^2 + 2y^2$.

1) 14.341

2) 7.90235

3) 2.61742

4) 3.07907

5) 14.2462

Exercise 2

Consider the vector field $F(x,y,z) = \left\{-x y + 6 z - \mathrm{Sin}[y^2],\ -7 y z - \mathrm{Sin}[2 x^2],\ e^{x^2-2y^2} - 7z + 7xz\right\}$ and the surface

$$S \equiv \left(\frac{-2+x}{6}\right)^2 + \left(\frac{-4+y}{1}\right)^2 + \left(\frac{z}{4}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −149.907 2) −902.407 3) 301.593 4) 90.8929

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -3(x-3)(x-2)x^2(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -\frac{x}{3} & 0 \le x \le 3 \\ \frac{x}{\pi-3} - \frac{3}{\pi-3} - 1 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$ and the moment $t = 0.9$ by means of a Fourier series of order 9 .

1) u(1 , 0.9) = −1.85304

2) u(1 , 0.9) = −1.17129

3) u(1 , 0.9) = 2.62175

4) u(1 , 0.9) = 1.25623

5) u(1 , 0.9) = 6.65067

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 80

Exercise 1

Given the function

$f(x,y,z) = -17 + 2x - x^2 + 6y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a minimum at {1, ?, 3}

2) We have a minimum at {?, -0.389117, -3.62958}

3) We have a minimum at {-0.0607348, ?, -3.32958}

4) We have a minimum at {?, -0.489117, -3.82958}

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-2v + 3v\,\mathrm{Cos}[u] - 2v\,\mathrm{Sin}[u],\ v - v\,\mathrm{Cos}[u] + v\,\mathrm{Sin}[u],\ v - v\,\mathrm{Cos}[u]\}$ at the point $(u,v) = (1, -7)$.

1) K (1 , -7) = 2.18177

2) K (1 , -7) = -0.896889

3) K (1 , -7) = -2.15228

4) K (1 , -7) = 5.07094

5) K (1 , -7) = 0

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2(x-3)(x-2)x(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2 and the moment t= 0.9 by means of a Fourier series of order 8 .

1) u(2 , 0.9) = -1.19571

2) u(2 , 0.9) = 4.63136

3) u(2 , 0.9) = 1.2121

4) u(2 , 0.9) = 1.00193

5) u(2 , 0.9) = 3.64092

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 81

Exercise 1

Compute the volume of the domain limited by the plane $4x + 6z == 10$ and the paraboloid $z == 3x^2 + 3y^2$.

1) 1.5198

2) 7.31337

3) 1.29952

4) 2.69468

5) 0.543516

Exercise 2

Consider the vector field $F(x,y,z) = \left\{e^{y^2-2z^2} + 6y - z,\ e^{-2z^2} - 3xy + 2yz,\ e^{-x^2+2y^2} + 5xyz\right\}$ and the surface

$$S \equiv \left(\frac{-3+x}{3}\right)^2 + \left(\frac{2+y}{4}\right)^2 + \left(\frac{-9+z}{6}\right)^2 = 1$$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) −22801.9 2) 16468.9 3) −6333.45 4) −5700.05

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -4x & 0 \le x \le 2 \\ 13x - 34 & 2 \le x \le 3 \\ -\frac{5x}{\pi-3} + \frac{15}{\pi-3} + 5 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} x & 0 \le x \le 2 \\ 16 - 7x & 2 \le x \le 3 \\ \frac{5x}{\pi-3} - \frac{15}{\pi-3} - 5 & 3 \le x \le \pi \end{cases} & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 2$ and the moment $t = 0.4$ by means of a Fourier series of order 9.

1) u(2 , 0.4) = 2.42697

2) u(2 , 0.4) = −3.26779

3) u(2 , 0.4) = 6.52387

4) u(2 , 0.4) = 8.92438

5) u(2 , 0.4) = −3.36682

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 82

Exercise 1

Compute the volume of the domain limited by the plane
$10x + 6z == 7$ and the paraboloid $z == 8x^2 + 8y^2$.

1) 0.967653

2) 0.308503

3) 0.233837

4) 1.2394

5) 0.128057

Exercise 2

Consider the vector field $F(x,y,z) =$
$\left\{e^{y^2+2z^2} + 6xy,\ xy - yz + \mathrm{Cos}\left[z^2\right],\ -7z - 5yz + \mathrm{Cos}\left[x^2 + 2y^2\right]\right\}$ and the surface

$S \equiv \left(\frac{-5+x}{7}\right)^2 + \left(\frac{-6+y}{8}\right)^2 + \left(\frac{4+z}{1}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −374.622 2) 1876.58 3) 7129.38 4) −4126.62

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = (x-3)^2 (x-2)(x-1)x^2 & 0 \le x \le 3 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -\frac{x}{2} & 0 \le x \le 2 \\ x-3 & 2 \le x \le 3 \end{cases} & 0. \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 2$
and the moment $t = 0.8$ by means of a Fourier series of order 9 .

1) u(2 , 0.8) = 0.660492

2) u(2 , 0.8) = 0.021232

3) u(2 , 0.8) = −7.19018

4) u(2 , 0.8) = 2.19637

5) u(2 , 0.8) = 2.17079

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 83

Exercise 1

Compute $\int_D (2x + z^3)\, dx\, dy\, dz$ for D=
$\{6x^3 \le y^6 z^4 \le 15x^3,\ 5 \le x^4 y^6 z^3 \le 10,\ 8x^5 y^5 \le z^4 \le 13 x^5 y^5,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.01945

2) 0.51945

3) 1.41945

4) 0.41945

5) 1.81945

Exercise 2

Compute the mean curvature for X(u,v) =
$\{2v + (1 + 2v^2)\,\mathrm{Cos}[u] + 2(1 + 2v^2)\,\mathrm{Sin}[u],\ v + (1 + 2v^2)\,\mathrm{Sin}[u],\ 3v + 2(1 + 2v^2)\,\mathrm{Sin}[u]\}$
at the point (u,v) = (4 , -6).

1) H (4 , -6) = -3.74962

2) H (4 , -6) = 0.000170781

3) H (4 , -6) = 5.15575

4) H (4 , -6) = -1.90582

5) H (4 , -6) = 5.61539

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 6x & 0 \le x \le 1 \\ x + 5 & 1 \le x \le 2 \\ -\frac{7x}{\pi-2} + \frac{14}{\pi-2} + 7 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.1 by means of a Fourier series of order 8 .

1) u(2 , 0.1) = 8.25674

2) u(2 , 0.1) = -2.08441

3) u(2 , 0.1) = 2.51927

4) u(2 , 0.1) = 5.20323

5) u(2 , 0.1) = -3.86948

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 84

Exercise 1

Given the system

$-3 x^2 y == 225$

$y^2 - 3 u_1^3 + 3 x u_3^2 == -51$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p = (x, y, u_1, u_2, u_3, u_4) = (-5, -3, 0, -4, -2, 4)$. Compute if possible $\frac{\partial y}{\partial u_4}(0, -4, -2, 4)$.

1) $\frac{\partial y}{\partial u_4}(0, -4, -2, 4) = 2$

2) $\frac{\partial y}{\partial u_4}(0, -4, -2, 4) = 3$

3) $\frac{\partial y}{\partial u_4}(0, -4, -2, 4) = 1$

4) $\frac{\partial y}{\partial u_4}(0, -4, -2, 4) = 0$

5) $\frac{\partial y}{\partial u_4}(0, -4, -2, 4) = 4$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-2 v - Cos[u], Sin[u], v + Cos[u] - 2 Sin[u]\}$ at the point $(u,v) = (6, 4)$.

1) $K(6, 4) = -1.97258$

2) $K(6, 4) = 0$

3) $K(6, 4) = -1.79916$

4) $K(6, 4) = -5.20398$

5) $K(6, 4) = -2.24175$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -3\,(x-3)\,(x-1)\,x^2\,(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.1 by means of a Fourier series of order 10 .

1) u(2 , 0.1) = 5.49318

2) u(2 , 0.1) = 4.12074

3) u(2 , 0.1) = -6.46945

4) u(2 , 0.1) = -7.49152

5) u(2 , 0.1) = 11.6008

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 85

Exercise 1

Given the system

$-2u^2 x + 3ux^2 + 2uy^2 - y^3 == 86$

$-u - 3u^2 x - 3v^2 x - 3uy - 3uvy == 34$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (-3 , 2 , 2 , 2). Compute if possible $\frac{\partial x}{\partial v}(2,2)$.

1) $\frac{\partial x}{\partial v}(2,2) = \frac{4}{37}$

2) $\frac{\partial x}{\partial v}(2,2) = \frac{8}{37}$

3) $\frac{\partial x}{\partial v}(2,2) = \frac{5}{37}$

4) $\frac{\partial x}{\partial v}(2,2) = \frac{6}{37}$

5) $\frac{\partial x}{\partial v}(2,2) = \frac{7}{37}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{u, -8u^2 - 4u(1+2v) + v(-7+9v), 8u^2 + (8-9v)v + u(4+8v)\}$ at the point $(u,v) = (-8, 8)$.

1) K (-8 , 8) = 0.908098

2) K (-8 , 8) = 6.18989

3) K (-8 , 8) = -4.98854×10^{-8}

4) K (-8 , 8) = 5.0941

5) K (-8 , 8) = 3.10511

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = -2\,(x-3)\,x^2\,(x-\pi) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$
and the moment $t = 0.3$ by means of a Fourier series of order 9 .

1) $u(2, 0.3) = -3.82503$

2) $u(2, 0.3) = 5.0941$

3) $u(2, 0.3) = -5.34232$

4) $u(2, 0.3) = -6.48054$

5) $u(2, 0.3) = 7.35419$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 86

Exercise 1

Compute $\int_D (y z^3)\,dx\,dy\,dz$ for $D=\{6 \le y^2 z^9 \le 14,\ 5 \le x^6 y^7 z^2 \le 9,\ 3 x^9 \le z^8 \le 10 x^9,\ x>0,\ y>0,\ z>0\}$

1) -0.895287

2) -0.595287

3) -0.295287

4) 0.00471286

5) 1.10471

Exercise 2

Compute the mean curvature for $X(u,v) = \{-4 - 2u^2 + 5u(-1+v) + 3v + 9v^2,\ v,\ -4 - 2u^2 + 4v + 9v^2 + u(-6+5v)\}$
at the point $(u,v) = (-1, 3)$.

1) H (-1 , 3) = 4.40624

2) H (-1 , 3) = 4.18965

3) H (-1 , 3) = -4.80809

4) H (-1 , 3) = 1.73075

5) H (-1 , 3) = 0.0457121

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -\frac{5x}{3} & 0 \le x \le 3 \\ \frac{5x}{2} - \frac{25}{2} & 3 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -6x & 0 \le x \le 1 \\ \frac{13x}{3} - \frac{31}{3} & 1 \le x \le 4 \\ 35 - 7x & 4 \le x \le 5 \end{cases} & 0. \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at x= 1
and the moment t= 1. by means of a Fourier series of order 8 .

1) u (1 , 1.) = 1.29408

2) u (1 , 1.) = -5.70046

3) u (1 , 1.) = 0.742814

4) u (1 , 1.) = -1.91423

5) u (1 , 1.) = -4.80809

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 87

Exercise 1

Compute the volume of the domain limited by the plane
$9x + z == 8$ and the paraboloid $z == 5x^2 + 5y^2$.

1) 219.842

2) 9.05625

3) 45.6167

4) 216.562

5) 13.9093

Exercise 2

Compute the area of the domain whose boundary is the curve

$r: [0, 2\pi] \dashrightarrow \mathbb{R}^2$

$$r(t) = \left\{ \frac{\left(-\frac{(\sqrt{3}-1)\sin(t)}{2\sqrt{2}} - \frac{1+\sqrt{3}}{2\sqrt{2}}\right)\cos(t)\,(\cos(t)+2)}{\sin^2(t)+1}, \frac{\left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{(1+\sqrt{3})\sin(t)}{2\sqrt{2}}\right)\cos(t)\,(\cos(t)+2)}{\sin^2(t)+1} \right\}$$

Indication: it is necessary to represent
the curve to check whether it has intersection points.

1) 7.25841   2) 5.25841   3) 4.85841   4) 8.05841

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ -\frac{7x}{\pi-2} + \frac{14}{\pi-2} + 7 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = -2(x-2)(x-1)x^2(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$
and the moment $t = 0.2$ by means of a Fourier series of order 12.

1) u(1 , 0.2) = −5.48098

2) u(1 , 0.2) = −0.191752

3) u(1 , 0.2) = −7.33746

4) u(1 , 0.2) = −7.5595

5) u(1 , 0.2) = 3.3182

Further Mathematics - Degree in Engineering - 2024/2025 Exam January-Call - computers for serial number: 88

Exercise 1

Given the function

$f(x,y,z) = 21 - 2x + x^2 - 4y + y^2 - 6z + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{16} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {-0.455515, -0.911031, ?}

2) We have a maximum at {?, -0.711031, -3.36239}

3) We have a maximum at {-0.655515, ?, -3.36239}

4) We have a maximum at {-0.655515, ?, -4.06239}

Exercise 2

Compute the Gauss curvature for X(u,v) = {Cos[u] (4 + 3 Cos[v]) + 2 (4 + 3 Cos[v]) Sin[u] - Sin[v], -2 Cos[u] (4 + 3 Cos[v]) - 3 (4 + 3 Cos[v]) Sin[u] + Sin[v], (4 + 3 Cos[v]) Sin[u]} at the point (u,v) = (5 , 5).

1) K (5 , 5) = 4.59147

2) K (5 , 5) = 0.0000689753

3) K (5 , 5) = 7.28138

4) K (5 , 5) = -6.98301

5) K (5 , 5) = -0.886884

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 7x & 0 \le x \le 1 \\ 12 - 5x & 1 \le x \le 3 \\ \frac{3x}{\pi-3} - \frac{9}{\pi-3} - 3 & 3 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1 and the moment t= 0.3 by means of a Fourier series of order 10 .

1) u (1 , 0.3) = -0.22569

2) u (1 , 0.3) = -0.236007

3) u (1 , 0.3) = 4.66014

4) u (1 , 0.3) = 2.41733

5) u (1 , 0.3) = 3.78426

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 89

Exercise 1

Given the system

$x\, y\, u_1 - 2\, x\, u_3 == -84$
$-2\, x\, y == -30$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-3, -5, -4, -2, -4, -4, 3)$. Compute if possible $\frac{\partial y}{\partial u_1}(-4, -2, -4, -4, 3)$.

1) $\frac{\partial y}{\partial u_1}(-4, -2, -4, -4, 3) = \frac{13}{4}$

2) $\frac{\partial y}{\partial u_1}(-4, -2, -4, -4, 3) = \frac{7}{2}$

3) $\frac{\partial y}{\partial u_1}(-4, -2, -4, -4, 3) = \frac{29}{8}$

4) $\frac{\partial y}{\partial u_1}(-4, -2, -4, -4, 3) = \frac{25}{8}$

5) $\frac{\partial y}{\partial u_1}(-4, -2, -4, -4, 3) = \frac{27}{8}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-18 u^2 - u (1 + 14 v) + 2 (-9 + 9 v + 8 v^2), v, -27 u^2 - u (2 + 21 v) + 3 (-9 + 9 v + 8 v^2)\}$ at the point $(u,v) = (-1, -1)$.

1) $K(-1, -1) = -0.947956$

2) $K(-1, -1) = 3.46071$

3) $K(-1, -1) = -5.63989 \times 10^{-6}$

4) $K(-1, -1) = -1.39941$

5) $K(-1, -1) = 2.23362$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t)=4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t)=u(\pi,t)=0 & 0\le t \\ u(x,0)=-2\,(x-3)\,(x-1)\,x\,(x-\pi) & 0\le x\le\pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 2
and the moment t= 0.1 by means of a Fourier series of order 12 .

1) u(2 , 0.1) = -3.94391

2) u(2 , 0.1) = -1.89389

3) u(2 , 0.1) = -4.38542

4) u(2 , 0.1) = 0.203108

5) u(2 , 0.1) = 3.46071

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 90

Exercise 1

Given the system

$3\,x\,y\,u_1 == -36$
$3\,u_2^2 - 2\,x\,u_1\,u_3 + 2\,y\,u_1\,u_5 == -18$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-1, -4, -3, 2, 1, 4, -1)$. Compute if possible $\frac{\partial y}{\partial u_3}(-3, 2, 1, 4, -1)$.

1) $\frac{\partial y}{\partial u_3}(-3, 2, 1, 4, -1) = \frac{4}{3}$

2) $\frac{\partial y}{\partial u_3}(-3, 2, 1, 4, -1) = \frac{7}{3}$

3) $\frac{\partial y}{\partial u_3}(-3, 2, 1, 4, -1) = \frac{5}{3}$

4) $\frac{\partial y}{\partial u_3}(-3, 2, 1, 4, -1) = \frac{8}{3}$

5) $\frac{\partial y}{\partial u_3}(-3, 2, 1, 4, -1) = 2$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{-4 + 4\,u^2 + u\,(7 - 8\,v) + 11\,v + 10\,v^2,\ v,\ -10 + 10\,u^2 + u\,(17 - 20\,v) + 27\,v + 25\,v^2\}$ at the point $(u,v) = (-5, -2)$.

1) $K(-5, -2) = -5.16742$

2) $K(-5, -2) = 4.9332$

3) $K(-5, -2) = -4.43383$

4) $K(-5, -2) = 4.40565 \times 10^{-6}$

5) $K(-5, -2) = -4.02895$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 6x & 0 \le x \le 1 \\ 8-2x & 1 \le x \le 2 \\ -\frac{4x}{\pi-2} + \frac{8}{\pi-2} + 4 & 2 \le x \le \pi \end{cases} & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x=1$ and the moment $t=0.4$ by means of a Fourier series of order 12 .

1) $u(1, 0.4) = -3.06591$

2) $u(1, 0.4) = 4.0958$

3) $u(1, 0.4) = 3.75701$

4) $u(1, 0.4) = 7.16202$

5) $u(1, 0.4) = 0.877452$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 91

Exercise 1

Compute the volume of the domain limited by the plane $x + 7z == 7$ and the paraboloid $z == 9x^2 + 9y^2$.

1) 0.103633

2) 0.174731

3) 0.721915

4) 0.117805

5) 0.234396

Exercise 2

Consider the vector field $F(x,y,z) = \{-2 + \text{Sin}[y^2 + z^2], 7xy + \text{Cos}[x^2 + z^2], -7yz + \text{Cos}[2y^2]\}$ and the surface

$$S \equiv \left(\frac{-3 + x}{7}\right)^2 + \left(\frac{6 + y}{9}\right)^2 + \left(\frac{-3 + z}{2}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 43 225.6 2) −59 849.4 3) 152 951. 4) 33 250.6

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 25\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t) = u(1,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{50x}{3} & 0 \le x \le \frac{3}{10} \\ 8 - 10x & \frac{3}{10} \le x \le \frac{7}{10} \\ \frac{10}{3} - \frac{10x}{3} & \frac{7}{10} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ \frac{\partial}{\partial t}u(x,0) = \begin{cases} -90x & 0 \le x \le \frac{1}{10} \\ 40x - 13 & \frac{1}{10} \le x \le \frac{1}{5} \\ \frac{25x}{4} - \frac{25}{4} & \frac{1}{5} \le x \le 1 \end{cases} & 0. \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = \frac{1}{2}$ and the moment $t = 0.3$ by means of a Fourier series of order 8.

1) $u\left(\frac{1}{2}, 0.3\right) = -3.401$

2) $u\left(\frac{1}{2}, 0.3\right) = 7.54686$

3) $u\left(\frac{1}{2}, 0.3\right) = 0.315821$

4) $u\left(\frac{1}{2}, 0.3\right) = -6.12027$

5) $u\left(\frac{1}{2}, 0.3\right) = -3.44016$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 92

Exercise 1

Compute $\int_D (x\, y)\, dx dy dz$ for D=

$\{4\, y \leq x\, z \leq 10\, y,\ 5\, x\, y\, z^4 \leq 1 \leq 10\, x\, y\, z^4,\ z^5 \leq x^8\, y^8 \leq 6\, z^5,\ x > 0,\ y > 0,\ z > 0\}$

1) −0.592898

2) 1.4071

3) 0.00710208

4) 1.9071

5) −0.792898

Exercise 2

Consider the vector field F(x,y,z) =

$\{8 - 4\, z - \text{Sin}[2\, y^2 - 2\, z^2],\ -10\, z + \text{Cos}[2\, x^2],\ e^{-2\, x^2 - y^2} + 8\, y + 6\, z\}$ and the surface

$S \equiv \left(\frac{-8 + x}{6}\right)^2 + \left(\frac{3 + y}{5}\right)^2 + \left(\frac{z}{8}\right)^2 = 1$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) 6031.86 2) 4825.66 3) 28 346.6 4) 13 872.2

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ u(0,t)=u(1,t)=0 & 0\le t \\ u(x,0)=\begin{cases} -\frac{50x}{7} & 0\le x\le \frac{7}{10} \\ \frac{50x}{3}-\frac{50}{3} & \frac{7}{10}\le x\le 1 \end{cases} & 0\le x\le 1 \\ \frac{\partial}{\partial t}u(x,0)=2\,(x-1)^2\left(x-\frac{3}{5}\right)\left(x-\frac{3}{10}\right)x^2 & 0.\le x\le 1 \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x=\frac{9}{10}$ and the moment $t=0.3$ by means of a Fourier series of order 12 .

1) $u\left(\frac{9}{10}, 0.3\right) = 2.10669$

2) $u\left(\frac{9}{10}, 0.3\right) = -3.68422$

3) $u\left(\frac{9}{10}, 0.3\right) = 4.21477$

4) $u\left(\frac{9}{10}, 0.3\right) = 0.728991$

5) $u\left(\frac{9}{10}, 0.3\right) = -0.102396$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 93

Exercise 1

Given the function

$f(x, y, z) = -27 + 4x - x^2 + 4y - y^2 + 6z - z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {1.7, ?, 3.6}

2) We have a maximum at {2, 2, ?}

3) We have a maximum at {?, 0.5, 1.8}

4) We have a maximum at {3.2, ?, 4.2}

Exercise 2

Consider the vectorial field $F(x, y, z) = (-2yz^2 \sin(xz) + 6xy + 4x, 3x^2 + 2z\cos(xz), 2y\cos(xz) - 2xyz\sin(xz))$. Compute the potential function for this field whose potential at the origin is -5.

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) 1.02636   2) 0.226364   3) −3.37364   4) 5.42636

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0 & 0 \le t \\ u(x,0) = -((x-1)\,x\,(x-\pi)) & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point x= 1
 and the moment t= 1. by means of a Fourier series of order 12 .

1) u(1, 1.) = −0.481591

2) u(1, 1.) = −2.00267

3) u(1, 1.) = −1.7043

4) u(1, 1.) = 0.938922

5) u(1, 1.) = −4.56143

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 94

Exercise 1

Compute $\int_D (2\, x^2)\, dx\, dy\, dz$ for D=

$\{2\, y^4 \leq x^7\, z^7 \leq 6\, y^4,\ 5\, x^2\, y^6 \leq z^3 \leq 12\, x^2\, y^6,\ 8\, z^4 \leq x^2\, y^9 \leq 9\, z^4,\ x > 0,\ y > 0,\ z > 0\}$

1) 0.0017793

2) −1.69822

3) 0.701779

4) 1.50178

5) −1.69822

Exercise 2

Consider the vector field $F(x,y,z) =$
$\{-5\, x\, y\, z + \mathrm{Cos}[z^2],\ -9\, y\, z + \mathrm{Cos}[x^2],\ x\, y + \mathrm{Sin}[2\, x^2 - 2\, y^2]\}$ and the surface

$S \equiv \left(\frac{9 + x}{8}\right)^2 + \left(\frac{-2 + y}{2}\right)^2 + \left(\frac{9 + z}{2}\right)^2 = 1$

Compute $\int_S F.$

Indication: Use Gauss' Theorem if it is necessary.

1) 22 921.1   2) −11 460.4   3) −36 673.5   4) 61 886.8

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 70x & 0 \le x \le \frac{1}{10} \\ \frac{70}{9} - \frac{70x}{9} & \frac{1}{10} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{4}{5}$ and the moment $t = 0.3$ by means of a Fourier series of order 9.

1) $u(\frac{4}{5}, 0.3) = 1.30389$

2) $u(\frac{4}{5}, 0.3) = -1.00439$

3) $u(\frac{4}{5}, 0.3) = 3.97317$

4) $u(\frac{4}{5}, 0.3) = 4.60781$

5) $u(\frac{4}{5}, 0.3) = 3.40028$

Further Mathematics - Degree in Engineering - 2024/2025
Exam January-Call - computers for serial number: 95

Exercise 1

Given the system

$-v x - 2 x^2 - u x^2 - 3 v y + u v y + x^2 y == -2$

$-x^2 + x^3 + 2 y + 2 x^2 y == 44$

determine if it is possible to solve for variables x , y in terms of variables u , v arround the point p = (x , y , u , v) = (-3 , 4 , 2 , 2) . Compute if possible $\frac{\partial y}{\partial u}(2,2)$.

1) $\frac{\partial y}{\partial u}(2,2) = \frac{3}{13}$

2) $\frac{\partial y}{\partial u}(2,2) = \frac{7}{13}$

3) $\frac{\partial y}{\partial u}(2,2) = \frac{4}{13}$

4) $\frac{\partial y}{\partial u}(2,2) = \frac{6}{13}$

5) $\frac{\partial y}{\partial u}(2,2) = \frac{5}{13}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{15 - 25 u + 6 u^2 - 21 v + 18 u v + 9 v^2, 10 - 17 u + 4 u^2 - 14 v + 12 u v + 6 v^2, -5 - 2 u^2 + u (8 - 6 v) + 8 v - 3 v^2\}$ at the point $(u,v) = (-8, 3)$.

1) $K(-8, 3) = -3.35617 \times 10^{-8}$

2) $K(-8, 3) = 6.85898$

3) $K(-8, 3) = 8.13066$

4) $K(-8, 3) = -8.06296$

5) $K(-8, 3) = 5.33138$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<3,\ 0<t \\ u(0,t) = u(3,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} -9x & 0 \le x \le 1 \\ \frac{9x}{2} - \frac{27}{2} & 1 \le x \le 3 \end{cases} & 0 \le x \le 3 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$ and the moment $t = 1.$ by means of a Fourier series of order 10 .

1) $u(2, 1.) = 8.61017$

2) $u(2, 1.) = -5.58586$

3) $u(2, 1.) = -8.04724$

4) $u(2, 1.) = 7.05353$

5) $u(2, 1.) = -0.076598$

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Exercise 1

Given the system

$3uvw - 2wx - 3vwy == 140$
$-u^2 - 3v^2x + 2vwy == -137$

determine if it is possible to solve for variables x , y in terms of variables u , v , w arround the point p=(x , y , u , v , w)=(4 , -2 , -5 , -4 , 5). Compute if possible $\frac{\partial x}{\partial v}(-5,-4,5)$.

1) $\frac{\partial x}{\partial v}(-5,-4,5) = \frac{36}{41}$

2) $\frac{\partial x}{\partial v}(-5,-4,5) = \frac{71}{82}$

3) $\frac{\partial x}{\partial v}(-5,-4,5) = \frac{73}{82}$

4) $\frac{\partial x}{\partial v}(-5,-4,5) = \frac{35}{41}$

5) $\frac{\partial x}{\partial v}(-5,-4,5) = \frac{69}{82}$

Exercise 2

Compute the Gauss curvature for X(u,v) = {Cos[u] (3 + Cos[v]), -4 (3 + Cos[v]) Sin[u] - 3 Sin[v], 7 (3 + Cos[v]) Sin[u] + 5 Sin[v]} at the point (u,v) = (5 , 6).

1) K (5 , 6) = 7.59522

2) K (5 , 6) = 8.53971

3) K (5 , 6) = -5.03187

4) K (5 , 6) = 0.00253011

5) K (5 , 6) = -3.19095

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<5,\ 0<t \\ u(0,t) = u(5,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} \frac{7x}{2} & 0 \le x \le 2 \\ 13 - 3x & 2 \le x \le 4 \\ 5 - x & 4 \le x \le 5 \end{cases} & 0 \le x \le 5 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 4$ and the moment $t = 0.7$ by means of a Fourier series of order 11 .

1) $u(4, 0.7) = -3.21514$

2) $u(4, 0.7) = -3.7045$

3) $u(4, 0.7) = 7.89733$

4) $u(4, 0.7) = -0.954883$

5) $u(4, 0.7) = 1.75647$

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Exercise 1

Given the system

$2xy^2 + yu_2 + 2u_3^2 - yu_4 - 2u_5 + 3xu_5 + 3xu_1u_5 == 12$
$3x^2y + 3x^2u_3 == -6$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4, u_5 arround the point $p = (x, y, u_1, u_2, u_3, u_4, u_5) = (-1, -4, -3, -4, 2, 3, 2)$. Compute if possible $\frac{\partial x}{\partial u_4}(-3, -4, 2, 3, 2)$.

1) $\frac{\partial x}{\partial u_4}(-3, -4, 2, 3, 2) = \frac{1}{2}$

2) $\frac{\partial x}{\partial u_4}(-3, -4, 2, 3, 2) = 1$

3) $\frac{\partial x}{\partial u_4}(-3, -4, 2, 3, 2) = \frac{5}{4}$

4) $\frac{\partial x}{\partial u_4}(-3, -4, 2, 3, 2) = \frac{1}{4}$

5) $\frac{\partial x}{\partial u_4}(-3, -4, 2, 3, 2) = \frac{3}{4}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{\text{Cos}[u](3 + 2\text{Cos}[v]), -2\text{Cos}[u](3 + 2\text{Cos}[v]) - (3 + 2\text{Cos}[v])\text{Sin}[u] - 2\text{Sin}[v], 2\text{Cos}[u](3 + 2\text{Cos}[v]) + (3 + 2\text{Cos}[v])\text{Sin}[u] + \text{Sin}[v]\}$ at the point $(u,v) = (1, 5)$.

1) $K(1, 5) = 8.25181$

2) $K(1, 5) = 8.26886$

3) $K(1, 5) = 1.91637$

4) $K(1, 5) = 0.00144444$

5) $K(1, 5) = -5.68685$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-2)\,x^2\,(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$
and the moment $t = 0.6$ by means of a Fourier series of order 11 .

1) $u(2, 0.6) = -5.99713$

2) $u(2, 0.6) = -2.4592$

3) $u(2, 0.6) = -2.25459$

4) $u(2, 0.6) = -1.9649$

5) $u(2, 0.6) = -0.19194$

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Exercise 1

Given the system

$-3\,x\,y - 2\,x\,u_1 + u_2\,u_3 + 3\,u_3^2 + 2\,x\,u_3\,u_4 == -25$

$3\,x\,y - u_1^2\,u_2 + 2\,y\,u_2\,u_4 == -1$

determine if it is possible to solve for variables x, y in terms of variables u_1, u_2, u_3, u_4 arround the point $p=(x, y, u_1, u_2, u_3, u_4) = (5, -1, 4, -1, 0, -1)$. Compute if possible $\frac{\partial x}{\partial u_1}(4, -1, 0, -1)$.

1) $\frac{\partial x}{\partial u_1}(4, -1, 0, -1) = -\frac{3}{13}$

2) $\frac{\partial x}{\partial u_1}(4, -1, 0, -1) = -\frac{2}{13}$

3) $\frac{\partial x}{\partial u_1}(4, -1, 0, -1) = -\frac{4}{13}$

4) $\frac{\partial x}{\partial u_1}(4, -1, 0, -1) = -\frac{5}{13}$

5) $\frac{\partial x}{\partial u_1}(4, -1, 0, -1) = -\frac{1}{13}$

Exercise 2

Compute the Gauss curvature for $X(u,v) = \{7\,u - 2\,v,\ -3\,u + v,\ 2 - 9\,u + 3\,u^2 - v - 3\,v^2\}$ at the point $(u,v) = (-8, -5)$.

1) $K(-8, -5) = -4.6256 \times 10^{-7}$

2) $K(-8, -5) = 5.85368$

3) $K(-8, -5) = -2.8972$

4) $K(-8, -5) = 7.15538$

5) $K(-8, -5) = 3.02617$

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-1)\,x\,(x-\pi)^2 & 0 \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = 2$
and the moment $t = 0.4$ by means of a Fourier series of order 8 .

1) $u(2, 0.4) = 1.10474$

2) $u(2, 0.4) = -3.75389$

3) $u(2, 0.4) = -0.858737$

4) $u(2, 0.4) = -1.00621$

5) $u(2, 0.4) = -4.44374$

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Exercise 1

Given the function

$f(x, y, z) = -3 + x^2 - 2y + y^2 + z^2$ defined over the domain D≡ $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{25} \leqslant 1$, compute its absolute maxima and minima.

1) We have a maximum at {0.2, -4.6, ?}

2) We have a maximum at {-0.2, ?, 0.2}

3) We have a maximum at {-0.1, -5.3, ?}

4) We have a maximum at {0., ?, 0.}

Exercise 2

Consider the vectorial field $F(x, y, z) = (y z \sin(x y z) + y z (x y z - 3 y z) \cos(x y z) - 2 x y + 2 x, -x^2 + (x z - 3 z) \sin(x y z) + x z (x y z - 3 y z) \cos(x y z), (x y - 3 y) \sin(x y z) + x y (x y z - 3 y z) \cos(x y z))$. Compute the potential function for this field whose potential at the origin is -5 .

. Calculate the integral of the potential function ϕ over the domain $[0,1]^3$.

1) -17.4594 2) -4.95936 3) -6.45936 4) -15.4594

Exercise 3

$$\begin{cases} \frac{\partial u}{\partial t}(x,t) = 16 \frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<1,\ 0<t \\ \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0 & 0 \le t \\ u(x,0) = \begin{cases} 40x & 0 \le x \le \frac{1}{5} \\ 10 - 10x & \frac{1}{5} \le x \le 1 \end{cases} & 0 \le x \le 1 \\ 0 & \text{True} \end{cases}$$

Compute the temperature of the bar at the point $x = \frac{1}{5}$ and the moment t = 0.5 by means of a Fourier series of order 9 .

1) $u(\frac{1}{5}, 0.5) = -2.39308$

2) $u(\frac{1}{5}, 0.5) = 4.75656$

3) $u(\frac{1}{5}, 0.5) = -2.99761$

4) $u(\frac{1}{5}, 0.5) = 4.$

5) $u(\frac{1}{5}, 0.5) = 3.03422$

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Exercise 1

Compute the volume of the domain limited by the plane $8x + 5z == 10$ and the paraboloid $z == 3x^2 + 3y^2$.

1) 12.0425

2) 0.280243

3) 4.27039

4) 11.5951

5) 2.56503

Exercise 2

Consider the vector field $F(x,y,z) = \left\{e^{2y^2+z^2} - 5x,\ 3z + \text{Cos}[x^2 - 2z^2],\ 9x + \text{Sin}[2x^2 - 2y^2]\right\}$ and the surface

$$S \equiv \left(\frac{-4+x}{4}\right)^2 + \left(\frac{-9+y}{5}\right)^2 + \left(\frac{1+z}{1}\right)^2 = 1$$

Compute $\int_S F$.

Indication: Use Gauss' Theorem if it is necessary.

1) −418.879   2) −544.579   3) −1675.88   4) −1508.28

Exercise 3

$$\begin{cases} \frac{\partial^2 u}{\partial t^2}(x,t) = 9\frac{\partial^2 u}{\partial x^2}(x,t) & 0<x<\pi,\ 0<t \\ u(0,t) = u(\pi,t) = 0 & 0 \le t \\ u(x,0) = (x-2)(x-1)x(x-\pi) & 0 \le x \le \pi \\ \frac{\partial}{\partial t}u(x,0) = 3(x-3)(x-1)x(x-\pi) & 0. \le x \le \pi \\ 0 & \text{True} \end{cases}$$

Compute the position of the string at $x = 1$ and the moment $t = 0.9$ by means of a Fourier series of order 8.

1) u(1, 0.9) = 2.29433

2) u(1, 0.9) = −3.78716

3) u(1, 0.9) = 1.26517

4) u(1, 0.9) = 4.55515

5) u(1, 0.9) = −3.73475